A Theory of Intensity, Electoral Competition, and Costly Political Action*

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Abstract: Individuals vary in how intensely they care about political outcomes. Despite attention to intensity in studies of representation and public opinion, the study of elections has paid less attention to the strategic dynamics of intensity. I present a theory that brings intensity to electoral competition. I investigate the pre-election actions of voters and the response of candidates through a game-theoretic model. Because intensity is unobserved and subject to misrepresentation, voters communicate intensity through costly political action. Candidates respond to voter actions by sometimes proposing policy opposed by a low-intensity majority. The theory suggests when and why citizens choose costly action and expression, describes why citizens might prefer candidates with negative traits such as history of misconduct, indicates when majoritarian systems might implement non-majoritarian policy, shows when costly political action is welfare-enhancing, and might help scholars reason about how candidates learn about voter interests.

Keywords: intensity of preference; electoral competition; representation; political action; asymmetric information; issue incongruence.

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Politics resolves differing values among members of a group. Group members might differ not only in desired result, but in how much they prefer ideal over alternatives, their *intensity of preference*. Scholars of representation have long argued that heterogeneity in intensity is significant for political settings with republican institutions. In *A Preface to Democratic Theory*, Robert Dahl (1956) opines on the "intensity problem" and attributes "one of the bloodiest civil wars in the history of Western man (98)" to intense preferences on the disposition of western lands. He concludes intensity a central challenge for a Madisonian republic. Fiorina (1974) proposes that representatives weight voting decisions by the intensities of different constituent groups and Fenno (1978) quotes one member of Congress, "There isn't one voter in 20,000 who knows my voting record . . . except on that one thing that affects him (142)."

Scholars of public opinion argue that differences in intensity structure how voters think about politics. Surveys suggest that citizens ascribe importance to or find salient at most a few policy issues (e.g., Krosnick, 1990; Rabinowitz and Macdonald, 1989) and are otherwise rather ambivalent, so-called "issue publics."

Despite consideration of intensity in studies of representation and opinion, many political science theories of electoral competition do not directly map intensity through the behavior of citizens to the response of candidates. Theories of elections focus either on how voter ideal points – setting aside differences in intensity for that ideal – structure policy competition between candidates or assume vote choice follows from non-policy psychological attachments or group identities. While either class of theory could in principle connect to heterogeneous intensity, in this essay I propose a theory that explicitly connects citizens' intensity to pre-election action, candidate policy proposals, and vote choice. Incorporating intensity to formal models of political action helps us understand why voters choose to take costly political action and expressions and why politicians sometimes propose policy with a known minority.

My argument is simply stated. Citizens vary in how intensely they care about policy and that intensity enters their choice between candidates. Because intensity influences vote choice, candidates respond to the distribution of intensity when proposing policy platforms. However, because individual intensity is not observed by others, those with intense preferences incur costs so that candidates know that they care intensely and others do not. Citizens might participate in politics in many ways, some of which are costly and might appear irrational or inconsistent with what is generally thought to be good democratic behavior. Even though these costly actions do not influence which candidate wins the election, some high-intensity voters engage in non-instrumental costly political action and expression to *communicate* intensity.

This "intensity theory" connects intensity, electoral competition, and the many different avenues citizens choose from to participate in politics with ideas from political economy on asymmetric information, mechanism design, costly signaling, and probabilistic voting. I formalize the argument in a game-theoretic model. In the model, citizens have different ideal policies and different intensities. Candidates know citizens' ideal policies but not their intensities. Candidates observe the costly political action chosen endogenously by each citizen. I model the action of majority and minority groups of voters separately, which can be of particular relevance when intensity is heterogeneous.[1]

The analysis presents three equilibria of interest. In all three equilibria, high-intensity voters choose political action and expression of personal cost because candidates learn information relevant to vote choice from action and expression. In all three equilibria, candidates propose policy with a high-intensity minority when that minority cares enough about the policy relative to the majority.

In a minority-only equilibrium, only voters in the minority who also care intensely engage in costly political action while voters in the majority always abstain. The magnitude of action required by the minority voter is relatively high compared to that in the other equilibria and is only supported when the ex ante beliefs that any individual voter cares

[1] Dahl noted the "crucial problem" when "the minority prefers its alternative much more passionately than the majority prefers a contrary alternative (90)."

intensely is relatively low.

In a separating equilibrium, both minority and majority voters who care intensely take costly action to communicate to candidates. The separating equilibrium is supported under a much wider range of ex ante beliefs about the rate voters care intensely on the issue than the range that supports the minority-only equilibrium. In both cases, the range of beliefs that supports equilibrium is increasing in the intensity of those who care most about the issue. Equilibrium magnitude of costly action is strictly less than in the minority-only equilibrium. I show that adding a second policy dimension to the electoral contest does not change results.

In an asymmetric equilibrium, only part of the majority joins the minority in choosing costly action when they care intensely. The other part of the majority never takes action. In such an equilibrium, acts of political participation would vary notably across the electorate. The asymmetric equilibrium only holds when intense preferences are ex ante believed to be relatively common. Equilibrium magnitude of action is again strictly less than equilibrium magnitude in the minority-only equilibrium and usually, but not always, greater than in the separating equilibrium.

Of particular note, in this equilibrium candidates believe part of the majority never takes costly action. These beliefs, however, do not mean the candidates do not represent the policy interests of this group. Instead, because candidates value the votes from any group, policy platforms are chosen based upon ex ante beliefs about the likelihood this group cares intensely about policy.

In a welfare analysis, I find that a system with costly political action and expression can improve electorate welfare relative to a system without such opportunities. When the minority cares more deeply about policy than the majority, costly political action causes candidates to propose minority policy against the known preferences of the majority. When intensity for policy is sufficiently strong, welfare for society in whole is improved because expected benefits to the minority of sometimes gaining non-majoritarian policy are greater than expected losses to the majority.[2] I compare both versions of electoral competition to mechanism design and find that, while a Vickrey-Clarke-Groves mechanism always chooses the efficient policy, in expectation it does non provide greater social welfare after subtracting transfers.

Although I interpret intensity theory in the context of citizens caring intensely about policy, the model can also speak to the dynamics of political identities. The key implications of the theory do not require public policy be the issue about which voters vary in intensity. Instead, the theory and model could have candidates choosing rhetoric and action that express social or political identities in response to voters taking action and expression to communicate how intensely they value expression of identity. Readers interested in such a setting might consider substituting "identity" for "policy" in the remainder of the essay to think about *identity intensity*. The theory can be read as response to the call by Achen and Bartels (2016, ch. 11) for more theory about political identities.

Incorporating intensity into formal models of political choice offers three substantive contributions to theories of elections. First, it suggests when and why voters with agency over their own behavior might choose to incur costs from activism, political participation, or pecuniary donations, or might choose to express statements of apparent bias towards political groups or inconsistent with democratic norms. This explanation does not require non-instrumental or intrinsic motivations for these actions.

Second, intensity theory brings vote choice together with pre-election political action and expressions into one model of electoral competition and communication. Vote choice and pre-election action are part of a portfolio of behavior rather than unitary decisions made independently.

Third, the theory provides new explanations for empirical patterns of politics. Intensity theory predicts that policy is sometimes proposed with the preferences of a minority of citizens even in a setting where everyone knows the preference of the majority and candidates

[2] This welfare analysis shows utilitarian but not Parieto improvement.

are motivated only to win votes. Intensity theory also challenges conventional interpretations of survey responses and implies that voters might sometimes prefer candidates with known negative traits simply because such candidates are costly to support.

Costly political behaviors should not always be taken as evidence that individuals derive immediate benefits from those behaviors or that action is motivated by non-instrumental duty, norms, or identities. If voters maintain support for unseemly candidates, parrot candidate propaganda, or falsely claim the economy faltered under the incumbent, we need not conclude that these actions or expressions are perceived as costless by the individuals who choose them. In fact, costliness might be what motivates the choice.

The model and theory connect to probabilistic models of elections and to mechanism design. Probabilistic models show that candidate policy depends on the distribution of voter utility functions. Mechanism design considers the difficulty of making efficient group decisions compatible with the incentives for individuals in the group. The challenge I address of communicating privately-held intensity of political preference is quite similar to the challenge of demand revelation in mechanism design theory.

The essay proceeds as follows. I first connect ideas from the political science of representation and intensity to existing explanations of costly political action and show how considering intensity and electoral competition generates an alternative interpretation of costly action. I then formalize the challenge faced by citizens and officials when intensity varies across individuals but is not observed into a game-theoretic model and present results. Following, I discuss how the three equilibria of interest add to our understanding of the political operation of representative democracy.

1 Intensity, electoral competition, and demand revelation

The importance of differing intensities for political outcomes is found in the political science of representation. In addition to Dahl (1956), Fenno (1978), and Fiorina (1974), intensity can be found in the Schattschneider (1960, ch. 2) discussion of pressure politics and the

Wilson (1995, ch. 16) discussion of concentrated benefits, distributed costs, and participation. Prior to presenting the spatial version of his theory, Downs (1957, ch. 3) argues voters consider the “expected party differential,” which depends on how much the voter cares for the platform of each party.

I understand intensity to mean how much a voter cares about one policy relative to how much they care about other political considerations. One could interpret intensity as the weight voters apply to that policy relative to valence issues. Importantly, this weight might be difficult for candidates to observe. While candidates in advanced democracies field surveys, hold focus groups, and meet directly with voters, voters might not have self-knowledge or incentive to accurately reveal their intensity (Dahl, 1956, p. 99-100). When equilibrium policy depends on the distribution of intensity in the electorate, a citizen with modest intensity might not accurately report their intensity if they know other voters might care more.

Formal-theoretic work in political economy provides ideas about how differences in intensity like those suggested by Dahl might influence elections and policy. Theories of probabilistic voting (e.g., Banks and Duggan, 2005; Coughlin and Nitzan, 1981; McKelvey and Patty, 2006; Persson and Tabellini, 2000) show that the policy candidates propose is a weighted sum of voter utility functions. When some voters have steeper utility functions with respect to policy, their vote choice is more responsive to policy than is the choice of voters with flatter utility functions. This means that candidates might be more responsive to the more intense voters than to less-intense voters.

While the importance of heterogeneous utility functions is understood in the formal political economy literature, it is less widely-appreciated by scholars of elections in political science. One goal of this paper is to bring this formal result to behavioral political science with a focus on the concept of “intensity.”

A second goal for this essay is to consider the consequences of intensity being hard for candidates to observe. I theorize that costly political action and expression serve as technology of costly communication. Individuals volunteer for campaigns, attend rallies, make political donations, sign petitions, write letters, call representatives, and attend public meetings. In addition to costly actions of political participation, many also make statements in public settings, in personal discussions, and in opinion surveys that appear obtuse if not inconsistent with objective evaluation of the political world. Some responses seem biased towards social or political groups in evaluations of candidates, policies, and matters of fact.

Most existing political science interprets costly actions and expressions through intrinsic psychological motivations rather than strategic consideration of the action's relationship to political outcomes. Textbooks, for example, suggest that participation in political activities follows not from "coldly rational" decision-making but from "[m]oral incentives, the personal satisfaction of active self-expression (Kernell et al., 2019, p 573)" or because "the political culture's emphasis on rights and liberties encourages Americans to contact their public officials and to protest government activities (Fiorina et al., 2011, p 175)."

While morality or culture might generate intrinsic motivations that explain costly political action, costly action can also arise in settings of strategic communication. I develop a model that builds upon other models of strategic communication over unobserved citizen preferences.[3] In the Lohmann (1993) model, citizens communicate about a common-value state parameter where the costly actions of some individuals are informative to political elites. The Meirowitz (2005) and Shotts (2006) models have voter ideal points private information. Pre-election communication occurs in Meirowitz through polls and in Shotts through votes at a first-stage election.[4]

Need to communicate private value for policy is also related to theories of mechanism design. Mechanism design deals with problems of collective choice where different actors in a group would gain different benefits from the collective action. For example, building a

[3] A set of studies consider settings where *politicians* possess knowledge that the voter does not (e.g. Ashworth and Bueno de Mesquita, 2014; Canes-Wrone, Herron, and Shotts, 2001; Fox and Van Weelden, 2015; Patty, 2016; Schnakenberg and Turner, 2019). See Gordon and Hafer (2005) for a model where corporations use contributions as communication.

[4] In legislative politics, Wawro and Schickler (2006, ch. 2) argue pre-1917 dilatory tactics in the U.S. Senate were costly behaviors used to communicate how intensely senators felt about the bill being obstructed.

bridge to connect two communities on each side of a river has commercial benefits for each community. However, one community might expect to benefit more than the other from the bridge. Both communities would like to minimize their own costly contribution to the construction of the bridge. Mechanism design uses the tools of game theory to consider different institutions to efficiently allocate costs between the parties.

Recent work has applied ideas from mechanism design to the challenges of differential benefits from public goods. Patty and Penn (2019) show that when voters must choose between candidates that each embody a platform of policy positions, the candidates elected might not correspond closely to the distribution of preferences in the population. Casella (2005) proposes a voting institution of *storable votes*, where members of a committee can abstain from voting on proposals they care less about to cast more votes on issues they care more about, allowing better reflection of intensity across issues. Lalley and Weyl (2018) advocate an institution of quadratic voting.

The analysis here takes as fixed the binary vote institution of representative democracy (like Patty and Penn, 2019) and asks when costly political action might be used as a mechanism to communicate demand for policy. This analysis adds to the current literature a direct focus on the agency of voters. It also considers explicitly how members of minorities and majorities on a policy issue respond differently with their costly action.

In the next section, I develop a game-theoretic model with four features. First, each voter's utility depends upon their individual intensity and policy preference. Second, electoral outcomes are influenced by the distribution of intensity in the electorate and the policies proposed by the candidates but not by the costly actions of voters. Third, voters have private knowledge of their own intensity. Fourth, each voter might choose a magnitude of costly political action to incur prior to candidate policy proposals that can communicate how much they care about that policy.

2 Strategic model of political action and intensity

To present the logic of intensity theory, I analyze a game-theoretic interaction between two candidates competing to win election before an electorate with heterogeneous ideal policies and heterogeneous intensity.

2.1 Primitives and payoffs

There are two candidates, A and B, and an electorate of three voters. Candidates are vote-maximizing and do not have preferences about policy.

Voters care about a binary policy s and are of preference $\tau = 0$ (preference-0) or $\tau = 1$ (preference-1) preferring $s = 0$ or $s = 1$. Voter policy preference is common knowledge for all voters and candidates. Assume that $\tau_i = 1$ for $i = \{1, 2\}$ and $\tau_3 = 0$ so $s = 0$ is the minority position. In addition to policy preference, voters vary in the *intensity* with which they care about the issue, $\beta_i \in \{1, \bar{\beta}\}$, $\bar{\beta} > 2$, representing low- and high-intensity. If policy is set at the voter's preference (e.g., $s = 0$ for a preference-0 voter), their payoff is β_i, $\bar{\beta}$ if high-intensity or 1 if low-intensity. Payoff is zero when policy is set opposite their preference. Intensity β_i is private knowledge for each voter i. However, the ex ante rate q that voters are high-intensity is common knowledge, $\Pr(\beta_i = \bar{\beta}) = q$ and $\Pr(\beta_i = 1) = 1 - q$, $q \in (0, 1)$.

2.2 Actions

Candidates each take one action, simultaneously proposing binding policy platforms s_A and $s_B \in \{0, 1\}$. Voters take two sequential actions. First, each voter chooses a magnitude of political actions of (net) cost $\lambda_i \in \mathbb{R}_+$. These actions are inherently costly in that the voter must pay immediate costs without certainty that they subsequently receive benefits that outweigh those costs. Through a diversity of available costly actions, voters choose continuous $\lambda_i > 0$ or choose no costly action, $\lambda_i = 0$.[5]

Second, voters cast a vote for one candidate given τ_i, β_i, s_A, and s_B. Vote choice is a ran-

[5]Voters have a variety of costly political behaviors of near-continuous intensity, for example monetary donations.

dom variable, represented in the model with an additive election-shock δ_i revealed at the time of the election. For simplicity, I assume each shock is drawn independently according to the uniform distribution with upper and lower bounds c and d common knowledge, though other distributions would not change the strategic incentives for candidates or voters. I assume $c < -\bar{\beta}$ and $\bar{\beta} < d$ so that the vote choice of all voters is stochastic.[6] Table 1 summarizes players, actions, and payoffs.

Table 1: Payoffs and Actions to the Game

Players	Voter 1	Voter 2	Voter 3	Candidate A	Candidate B
Actions	λ_1	λ_2	λ_3	s_A	s_B
Payoffs, A wins:	$\beta_1 s_A - \lambda_1$	$\beta_2 s_A - \lambda_2$	$\beta_3(1 - s_A) - \lambda_3$	Votes for A	Votes for B
Payoffs, B wins:	$\beta_1 s_B - \lambda_1$	$\beta_2 s_B - \lambda_2$	$\beta_3(1 - s_B) - \lambda_3$	Votes for A	Votes for B

2.3 Timing

1. Nature independently draws each β_i, $i \in 1, 2, 3$ from $\{1, \bar{\beta}\}$ given q.
2. Voters privately observe β_i and then simultaneously choose pre-election actions λ_i.
3. Candidates observe $\{\lambda_1, \lambda_2, \lambda_3\}$ then propose policy platforms s_A and s_B.
4. Nature independently draws each δ_i, election held, and votes realized. Candidate with majority wins election.
5. Payoffs realized.

2.4 Strategies and beliefs

At the election, voter choice follows a weakly dominant strategy to select the candidate proposing the preferred platform. Vote choice itself is stochastic.[7] I therefore focus on

[6] I show in Appendix Section B that the results of the paper remain when $-\bar{\beta} < c < -2$ and $2 < d < \bar{\beta}$. At these parameter values, vote choice of high-intensity types is degenerate but vote of low-intensity remains stochastic.

[7] Assume if indifferent, choose A with probability 0.5 and B with probability 0.5.

voter strategies over λ, which is a function $\sigma_V(\beta_i, \tau_i) : \{1, \bar{\beta}\} \times \{0, 1\} \to \mathbb{R}_+$ mapping intensity and policy preference into political action λ_i.

Define $\lambda \equiv (\lambda_1, \lambda_2, \lambda_3)$ and $\beta \equiv (\beta_1, \beta_1, \beta_1)$. For both candidates, a strategy is a function $\sigma_p(\lambda) : \mathbb{R}^3_+ \to \{0, 1\},\ p \in \{A, B\}$, mapping observed political actions into a policy platform s_p. Candidate beliefs depend upon observation of λ because they do not observe intensities β and vote totals depends on intensity. Candidates learn about intensity by the costly actions taken by each voter and use Bayes' Rule to update beliefs about β. For party $p \in \{A, B\}$ beliefs are

$$h_p : \mathbb{R}^3_+ \to \Delta(\{1, \bar{\beta}\}^3), \tag{1}$$

where $\Delta(\{1, \bar{\beta}\}^3)$ is the set of lotteries over voter intensities. There is no asymmetric information and so candidate beliefs are equivalent.

I use Perfect Bayesian Equilibrium (PBE) as solution concept. For a PBE, each candidate's policy strategy must be a best response given the other candidate's policy strategy and candidate beliefs about β. Candidate beliefs are consistent and updated by Bayes' Rule. The voter strategy must be a best response given candidate strategies and beliefs and that other voters are also playing best responses. I focus on equilibria in pure strategies.

3 Candidate best responses

Lemma 1 shows that, for both candidates, the best response is to propose $s_p = 0$ when $\hat{\beta}_1 + \hat{\beta}_2 \leq \hat{\beta}_3$ else $s_p = 1$, where $\hat{\beta}_i$ is the candidates' belief about the intensity of Voter i after observing λ_i.

Lemma 1 (Candidate best responses). *When the support of election shock δ, $[c, d]$, includes the values $-\bar{\beta}$ and $\bar{\beta}$, the best response to beliefs $\{\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3\}$ for both candidates is to propose the policy preferred by minority Voter 3, $s^* = 0$, when $\hat{\beta}_1 + \hat{\beta}_2 \leq \hat{\beta}_3$, otherwise to propose the policy preferred by majority Voters 1 and 2, $s^* = 1$.*

Proof. See Appendix Section A. □

Lemma 1 clarifies when candidates' electoral goal of maximizing votes is optimized by proposing policy with the majority and when optimized by proposing policy with the minority. The lemma states that the candidates are better off siding with the minority when the minority's intensity is larger than the sum of the intensities of the two majority voters. In this setting, candidate total votes are more responsive to the policy preferences of the intense minority voter than to the policy preferences of the less-intense majority voter.

The result depends on vote choice being stochastic through the election shocks δ. Lemma 1 assumes that the range of the δ distribution is greater than the magnitude of intensity of high-intensity voters such that, even for high-intensity voters, there is some probability that they will vote against the candidate proposing their preferred policy. I show, however, in Lemma A1 (Appendix Section B) that the best responses described by Lemma 1 are also best responses when $-\bar{\beta} < c < -2$ and $2 < d < \bar{\beta}$. As long as low-intensity voter choice remains stochastic, the electoral incentives for the candidates described in Lemma 1 remain.

The substantive importance of Lemma 1 is that candidates pursuing only votes – i.e., no personal preferences for policy or quid pro quo corruption – sometimes side with a high-intensity minority over a low-intensity majority. They do so when the minority is sufficiently more intense than the majority, in this case when the minority's intensity is larger than the sum of the intensities of the two majority voters.

4 Minority representation through political action

I turn now to describing three equilibria of interest for intensity theory. The first analysis shows that an equilibrium exists where a high-intensity minority communicates preferences through costly political action while both low- and high-intensity majority voters abstain from action. In this equilibrium, candidates propose policy with the minority when the minority chooses costly action and propose policy with the majority when the minority abstains from action. Voter 3's costly political action informs the candidates he or she is high-intensity, which induces the candidates to propose equilibrium policy platforms $s_A^* = s_B^* = 0$ following their best responses in Lemma 1. This result is stated in Proposition 1:

Proposition 1 (Minority policy representation through political action)**.** *For* $q < 1 - \sqrt{2}/2$ *and* $1 \leq \lambda^* \leq \bar{\beta}$*, there exists* $B > 2$ *such that* $\bar{\beta} \geq B$ *implies that there exists an equilibrium in which Voter 3, and only Voter 3, chooses costly action when high-intensity and abstains from costly action when low-intensity. In this equilibrium, the players' strategies are*

$$\sigma_1^*(1) = \sigma_2^*(1) = \sigma_1^*(\bar{\beta}) = \sigma_2^*(\bar{\beta}) = 0,$$

$$\sigma_3^*(1) = 0,\ \sigma_3^*(\bar{\beta}) = \lambda^*,$$

$$\sigma_A^*(\lambda) = \sigma_B^*(\lambda) = \begin{cases} 0 & \textit{if } \lambda_3 = \lambda^* \\ 1 & \textit{if } \lambda_3 = 0. \end{cases}$$

Furthermore, candidate beliefs $h_A^*(\lambda)$ *and* $h_B^*(\lambda)$*,* $\lambda \equiv (\lambda_1, \lambda_2, \lambda_3)$*, assign probability 1 to* $\beta_3 = 1$ *when* $\lambda_3 < \lambda^*$ *and probability 1 to* $\beta_3 = \bar{\beta}$ *when* $\lambda_3 \geq \lambda^*$*. Regardless of* λ*,* $h_A^*(\lambda)$ *and* $h_B^*(\lambda)$ *assign probability* q *to* $\beta_1 = \bar{\beta}$*, probability* q *to* $\beta_2 = \bar{\beta}$*, probability* $1 - q$ *to* $\beta_1 = 1$*, and probability* $1 - q$ *to* $\beta_2 = 1$*. Finally, only the equilibrium where* $\lambda^* = 1$ *is intuitive.*

Proof. See Appendix Section C. □

Voter 3 can only gain policy representation by convincing the candidates of their intensity. They choose to do so through costly political action when the expected benefits to obtaining representation outweigh the costs of the action.[8] This generates the parameters of Proposition 1.

Three results from Proposition 1 are of substantive political importance. First, in this equilibrium only voters who both care intensely and are on the minority side of the issue choose to engage in costly political action. Voters in the majority benefit enough, on average, without action to make incurring the costs of action undesirable. In such an equilib-

[8] I show in Appendix Section I that there is an equilibrium where voters choose costly political action when policy preference is evenly split in the electorate, i.e. no majority or minority.

rium, society would observe costly political action taken only by intense members of policy minorities.

Second, the magnitude of action required by the minority voter is relatively high, with a minimum cost of one equal to the value low-intensity voters gain from policy. A voter who incurs a cost equal or greater than the benefit of a low-intensity voter communicates to candidates that they can only be high-intensity. This equilibrium is relatively costly for the intense minority voter.

Third, this equilibrium is only supported when the ex ante beliefs that any individual voter cares intensely is relatively low. The ex ante rate high-intensity must be lower than $1 - \sqrt{2}/2$, which is about 0.3. Intensity must be relatively uncommon to support an equilibrium where only minority voters incur costs of political action.

5 Representation through separating political action

In a second equilibrium, majority voters join the minority in choosing costly action when high-intensity. Majority Voters 1 and 2 have different considerations than Voter 3 because there are three different pathways for them to attain their desired policy. Voter 1 (resp. 2) gains $s = 1$ either when Voter 1 (resp. 2) chooses costly action, when the other member of the majority Voter 2 (resp. 1) chooses costly action, or when all three voters abstain from costly action. Majority voters also know that when q is near one or near zero, majority policy is very likely to be proposed because, when near one, a majority voter is likely to be high-intensity with probability approaching one and, when near zero, the minority voter is likely to be low-intensity with probability approaching one. In the three-voter model here, "near zero" is less than $1/(\bar{\beta} + 1)$ and "near one" is greater than $\bar{\beta}/(\bar{\beta} + 1)$, but these values would differ for different balances of majority and minority.

In the separating equilibrium, each voter chooses to incur costly political action when high-intensity and to abstain when low-intensity. Costly action communicates intensity to candidates. Candidates propose equilibrium policy platforms $s_A^* = s_B^* = 0$ if and only if both majority voters abstain and the minority voter takes action. I present this result in Proposition 2:

Proposition 2 (Representation through separating political action)**.** *For any* $q \in (1/(\bar{\beta}+1), \bar{\beta}/(\bar{\beta}+1))$, *there exists* $B > 2$ *such that* $\bar{\beta} \geq B$ *implies that there exists a separating equilibrium. In this equilibrium, the players' strategies are*

$$\sigma_i^*(1) = 0, \ \sigma_i^*(\bar{\beta}) = \lambda^*, \ i \in \{1,2,3\}$$
$$\sigma_A^*(\lambda) = \sigma_B^*(\lambda) = \begin{cases} 0 & \textit{if } \lambda_1 = \lambda_2 = 0, \lambda_3 = \lambda^* \\ 1 & \textit{otherwise.} \end{cases}$$

and λ^* *respects the bounds*

$$(1-q)^2 \leq \lambda^* \leq \bar{\beta}(1-q-(1-q)^2) \textit{ if } q < 1/2,$$
$$1-q-(1-q)^2 \leq \lambda^* \leq \bar{\beta}(1-q)^2 \textit{ if } q \geq 1/2.$$

Furthermore, candidate beliefs $h_A^*(\lambda)$ *and* $h_B^*(\lambda)$, $\lambda \equiv (\lambda_1, \lambda_2, \lambda_3)$, *assign probability 1 to* $\beta_i = 1$ *when* $\lambda_i < \lambda^*$ *and probability 1 to* $\beta_i = \bar{\beta}$ *when* $\lambda_i \geq \lambda^*$ *for all* $i \in \{1,2,3\}$. *Finally, only the equilibrium where* λ^* *equals* $(1-q)^2$ *when* $q < 1/2$ *and* $1-q-(1-q)^2$ *when* $q \geq 1/2$ *is intuitive.*

Proof. See Appendix Section D. □

To provide intuition for this equilibrium, consider the following parameters: $\bar{\beta} = 6, q = 0.25, \lambda^* = 0.5625, c = -10, d = 10$.[9] Would any voter benefit deviating from the equilibrium? Start with Voter 3. A high-intensity Voter 3's expected benefit when choosing costly action is $(1-q)^2(\bar{\beta}) - \lambda^*$. The first term is the probability that Voters 1 and 2 are both low-intensity times the benefit to Voter 3 when $s = 0$. The second term is the costs of political action paid regardless of candidate policy.

[9] It is not necessary for the uniform distribution to be symmetric about zero.

At these parameter values, Voter 3's expected benefit is 2.81. If Voter 3 were to deviate from this equilibrium and not take costly action even when high-intensity, $s^* = 1$ with certainty because $\hat{\beta}_1 + \hat{\beta}_2 > \hat{\beta}_3$ when $\hat{\beta}_3 = 1$. As $2.81 > 0$, Voter 3 is better off not deviating. Continuing with Voters 1 and 2, whenever either plays λ^*, $s^* = 1$. The benefit to a high-intensity Voter 1 or 2 who chooses λ^* in the separating equilibrium is $\bar{\beta} - \lambda^* = 5.4$. If one of the two majority voters deviates and abstains from costly action, their expected benefit depends on whether or not Voter 3 is high-intensity (probability q) and whether or not the other majority voter is high-intensity, $q(1 - q) * 0 + (1 - q) * \bar{\beta} = 4.5$. As $5.4 > 4.5$, neither majority voter benefits from deviating.

This example shows the logic that supports the separating equilibrium. Each player chooses to incur costly political action when high-intensity to increase the probability (guarantee in the case of a majority voter) that policy is implemented at their preference. A minority voter makes these choices when the benefit to policy is sufficiently high to merit the risk that they will pay the costs of political action but not gain policy benefits. A majority voter makes the same choice while also weighing the probability that another member of the majority takes costly action or that the minority voter does not take action. Aggregate welfare is higher with political action in this example. With the separating strategies, expected benefits are $2.81 + 2 * 5.015 = 12.8$ versus without $0 + 2 * 6 = 12$. I provide a proof that (utilitarian) welfare benefits to costly communication hold more generally below.

Three results from Proposition 2 are of substantive political importance. First, in this equilibrium high-intensity voters on both sides of the issue take action to communicate to candidates. In such a setting, we would observe voters of many different policy views taking action to inform candidates how much they care about the issue.

Second, the equilibrium is supported under a much wider range of ex ante beliefs about the rate voters care intensely on the issue. While under Proposition 1 q must be less than about 0.3, in Proposition 2 $q \in (1/(\bar{\beta} + 1), \ \bar{\beta}/(\bar{\beta} + 1))$. These bounds show that the more those with high-intensity care about the issue, the wider the range of ex ante beliefs q can support a separating equilibrium.

Third, equilibrium magnitude of costly action λ^* in Proposition 2 is strictly less than that of Proposition 1 (see Appendix Section G). In an equilibrium of costly action by both majority and minority, the level of costly political action incurred by those who care intensely is lower than that incurred in an equilibrium where only the minority takes action. While we might see more voters taking action, the costliness of the actions they take is lower.

6 Divided majority: Asymmetric action equilibrium

In a third equilibrium, only one of the two majority voters join the minority in choosing costly action when high-intensity. The other majority voter commits to a never-action strategy, abstaining from action even if high-intensity. The equilibrium is maintained because the never-action majority voter gains policy enough in expectation without taking action. The never-action voter gains desired policy with probability $q + (1 - q)^2$ when the other majority voter takes action (probability q) or when neither other voter take action ($(1 - q)^2$). The majority voter who takes separating action does so when high-intensity to induce the candidates to propose policy with the majority knowing the other member of their policy coalition will abstain.

I assume, without loss of generality, that Voter 1 is the never-action majority voter and Voter 2 takes action when high-intensity. In equilibrium, costly action by Voters 2 and 3 communicates intensity to candidates. Candidates propose equilibrium policy platforms $s_A^* = s_B^* = 0$ if and only if Voter 2 abstains from action and Voter 3 takes action, otherwise $s_A^* = s_B^* = 1$. I present this result in Proposition 3:

Proposition 3 (Representation through asymmetric majority political action). *For any* $1/2 < q < \bar{\beta}/(\bar{\beta} + 1)$, *there exists* $B > 2$ *such that* $\bar{\beta} \geq B$ *implies that there exists a separating* *equilibrium. In this equilibrium, the players' strategies are*

$$\sigma_1^*(1) = \sigma_1^*(\bar{\beta}) = 0,$$

$$\sigma_i^*(1) = 0,\ \sigma_i^*(\bar{\beta}) = \lambda^*,\ i \in \{2, 3\}$$

$$\sigma_A^*(\lambda) = \sigma_B^*(\lambda) = \begin{cases} 0 & \textit{if } \lambda_3 \geq \lambda^*, \lambda_2 < \lambda^* \\ 1 & \textit{otherwise}. \end{cases}$$

and λ^* *respects the bounds* $q \leq \lambda^* \leq (1-q)\bar{\beta}$.

Furthermore, candidate beliefs $h_A^*(\lambda)$ *and* $h_B^*(\lambda)$, $\lambda \equiv (\lambda_1, \lambda_2, \lambda_3)$, *assign probability 1 to* $\beta_3 = 1$ *when* $\lambda_3 < \lambda^*$, *probability 1 to* $\beta_3 = \bar{\beta}$ *when* $\lambda_3 \geq \lambda^*$, *probability 1 to* $\beta_2 = 1$ *when* $\lambda_2 < \lambda^*$, *and probability 1 to* $\beta_2 = \bar{\beta}$ *when* $\lambda_2 \geq \lambda^*$. *Regardless of* λ, $h_A^*(\lambda)$ *and* $h_B^*(\lambda)$ *assign probability* q *to* $\beta_1 = \bar{\beta}$ *and probability* $1-q$ *to* $\beta_1 = 1$. *Finally, only the equilibrium where* $\lambda^* = q$ *is intuitive.*

Proof. See Appendix Section E. □

Four results from Proposition 3 are of substantive political importance. First, in this equilibrium the minority chooses costly action when high-intensity but only part of the majority takes action when high-intensity. The other member of the majority never takes costly action. In such an equilibrium, we see some members of the majority who (sometimes) take actions that suggest they care intensely while other members of the majority never take action even if they care intensely. This would be a setting where acts of political participation varied notably across the electorate.

Second, to support this equilibrium the ex ante beliefs that any individual voter is high-intensity must be relatively large ($q > 1/2$). Asymmetric action only holds when intense preferences are believed to be more common than not.

Third, equilibrium magnitude of costly action λ^* is strictly less than the equilibrium magnitude in the minority-only equilibrium. The magnitude λ^* is strictly greater than that in the separating equilibrium when $q < (3-\sqrt{5})/2$ or $q \geq 1/2$ but less when $(3-\sqrt{5})/2 < q < 1/2$

(see Appendix Section G). Voters in this equilibrium incur more costly action than they do in the separating equilibrium where all voters take action when high-intensity at most values of q, but less costly action than in the minority-only equilibrium at all values of q.

Fourth, in this equilibrium candidates believe part of the majority never takes costly action. This, however, does not mean the candidates do not represent the policy interests of this group. Instead, candidates propose policy based upon the ex ante beliefs q this voter is high-intensity. The other member of the majority coalition, however, must act to communicate to the candidates that they care intensely about the issue.

Candidate beliefs about the relationship between intensity and action for different voters in this equilibrium have consequences for how voter policy preferences are reflected in candidate platforms.

The PBE solution concept supports a continuum of equilibria. I have focused on three because they are of particular political interest. However, it is important to note there are other equilibria, including an equilibrium where λ^* is large enough that even high-intensity minority voters do not benefit from incurring costly political action. This equilibrium (along with other babbling equilibria) would have candidate strategies independent of voter action λ and voter strategies independent of voter intensity.

7 Robustness and extensions to equilibria

Two dimensions of policy

In Appendix Section F, I present an extension to the model where voters value and candidates propose two policies. This allows me to show, first, that results hold in the presence of a second dimension of policy contestation. Second, I consider homogeneous intensity on the second dimension so that the extension allows an exploration of how intensity influences proposals. I show that without intensity, both candidates' dominant strategy is to propose policy with the majority. That is, there are equilibria where policy is sometimes proposed with the minority on one dimension due to heterogeneous intensity but on the other dimension always proposed with the majority because voters do not vary in intensity. Heterogeneous intensity is a necessary condition to generate non-majoritarian policy.

Incentive to shirk in a large electorate?

In this simple model, the choices of minority voters are pivotal when the majority is low-intensity. In a larger electorate, the actions of individual voters are very unlikely to be pivotal and they therefore have incentive to shirk, i.e. choose $\lambda_i = 0$ even if high-intensity. The equilibrium can be retained with a large electorate with an additional assumption about candidate beliefs. Assume candidates infer high-intensity of a group of voters when $\lambda_i = \lambda^*$ for all i in that group. If any voter i chooses $\lambda_i = 0$, candidates believe that group is low-intensity and set policy accordingly. In other words, under such an assumption each member of a high-intensity group is pivotal in determining candidate beliefs and therefore pivotal to expected policy payoffs.[10]

Note, too, that the magnitude of λ^* is endogenous to the size of the electorate, declining in the number of voters. This may be one way to interpret low levels of political participation generally observed. We do not observe political contributors bankrupting themselves making political donations nor campaign volunteers working to prostration. Rather, even campaign volunteers and donors devote generally modest efforts to costly political action.

8 Costly political action increases welfare

In this section, I show that costly political action and expression can increase social welfare. While the majority is strictly worse off when the minority has access to costly communication, I show that when policy benefits to high-intensity types are sufficiently large, expected net gains to the minority exceed expected net losses to the majority.

I present the full welfare analysis in Appendix Section H. I define voter benefit from policy as a function $v_i(s, \beta_i)$ that depends upon the voter's policy preference τ_i and intensity β_i. I then define total voter welfare as their benefit to policy less any costs from political

[10] An alternative solution is that of Lohmann (1993) where private signals are correlated so that individual action can be pivotal to candidate beliefs.

action (λ_i) or transfers (t_i) assigned by the social planner in a mechanism design, $w_i = v_i - \lambda_i - t_i$. Social welfare is the sum over the voters, $W = \sum_i w_i$.

I calculate the expected value of W in three settings of policy choice. Settings 1 and 2 chooses policy through electoral competition as in the model presented above. In Setting 1, candidate strategies are independent of voter actions λ and voter strategies are independent of voter intensity. Candidates choose policy based only on prior beliefs about intensity (the rate q). Setting 2 analyzes the separating equilibrium in Proposition 2.

Setting 3 substitutes a Vickrey-Clarke-Groves (VCG) mechanism for electoral competition to allocate policy. VCG mechanisms select the efficient policy to maximize policy welfare but are not always budget-balanced. The VCG mechanism serves as an interesting benchmark against which to compare costly signaling in electoral competition. In fact, the electoral institutions are social choice mechanisms of a different flavor.

Welfare is improved when voters have the opportunity to engage in costly political action when the policy benefit for high-intensity voters is sufficiently large, as stated in Proposition 4:

Proposition 4 (Social welfare with costly political action). *The separating equilibrium from Proposition 2 leads to higher expected electorate welfare than in a setting without costly political action if and only if $\bar{\beta} > 5$.*

Both settings of electoral competition, with or without costly political action, produce greater welfare than a Vickrey-Clarke-Groves mechanism used to determine policy.

Proof. See Appendix Section H. □

Under Proposition 4, the minority benefits from costly communication because without communication the minority never attains preferred policy. Expected welfare for the majority, however, is strictly lower with communication. The majority loses from communication through two channels. First, majority voters must sometimes take costly political action when high-intensity, costs they need not incur in the no-communication setting.

Second, policy is sometimes set to the preference of the minority when it would not be in the no-communication setting.

Expected losses to the majority are outweighed by expected gains to the minority when $\bar{\beta} > 5$. In other words, when intense voters care sufficiently more about the issue than the less-intense, social welfare W is improved with costly communication because policy is more likely set with those who care most.

VCG mechanisms also set policy with those who care most. However, the mechanism does so by allocating transfers to induce voters to accurately report how much they care. While these transfers lead VCG to implement the efficient policy at every information set (not always the case in the two settings of electoral competition), in expectation these transfers lead to lower (utilitarian) social welfare than in the two settings of electoral competition.

8.1 Wealth, resources, participation, and welfare

One claim about political participation due to Schattschneider (1960) and others is that participation in politics tends to be more common among those with higher socio-economic status. This is clearly the case for campaign donations, which require sufficient disposable wealth. Other forms of participation are also empirically related to wealth and education. Readers might be concerned that the welfare of the well-resourced is favored by this analysis because the model and welfare analysis do not differentiate the resources available to different voters.

The welfare analysis under reasonable assumptions does not favor voters with greater resources. The magnitude λ^* represents the net costs a voter needs to incur to signal high-intensity. While these costs are incurred through specific political acts, the acts must be chosen such that the net costs *to the individual* are of magnitude λ^*.

It need not be the case that the same acts have the same net costs for each voter. Imagine extending the model so that λ_i was a function of the set of actions chosen, x, and features of the individual i, $\lambda_i = \ell_i(x)$. With diminishing marginal utility to wealth, the action x = $\{$donating \$1,000$\}$ would incur a different magnitude of cost λ for wealthy voter i than for impoverished voter i$'$, $\ell_i(x) < \ell_{i'}(x)$. A \$1,000 donation from a billionaire has different consequences for the billionaire than for the voter living paycheck-to-paycheck. A similar logic extends to those with more or less free time, those with more or less education, those with more or less political connections, etc. In order to incur the net cost λ^*, the well-resourced have to take more costly political action to counteract their greater resources.

9 Discussion: Empirical implications for electoral behavior

The previous sections explored a game-theoretic representation of intensity theory. The analysis presented three equilibria of political interest, each with interesting implications. In each equilibrium, policy is sometimes proposed with an intense minority when costly action communicates to the candidates that the minority cares more, or is more likely to care more, about policy than the majority. In the minority-only equilibrium, we observe costly political action only among members of a policy minority, with the majority abstaining from action even when they care intensely about the issue. In the separating equilibrium, voters who care intensely about the issue engage in costly action whether members of the majority or minority viewpoint. And in the asymmetric equilibrium, some voters in the majority follow a separating strategy where they engage in costly action when caring intensely about policy and abstain when caring only modestly while other voters in the majority abstain from action regardless of how much they care about the issue.

These results might help us understand a set of empirical regularities from electoral politics. Most directly, the theory highlights why voters vary in their level of costly political engagement. Some knock on doors, make campaign contributions, attend public meetings, or volunteer for political causes while others do not. In these equilibria, when these actions are costly, they serve to communicate intensity.

The asymmetric majority equilibrium provides another explanation of differences in political participation. If candidates believe that some voters take or abstain from action for reasons other than that they care intensely about policy while other voters take action only when they care intensely about policy, incentives to take action for voters vary substantially. The first group of voters might always or never engage in action, while the second group engages sometimes, only if they care intensely about the issue. This would generate variation cross-sectionally or over time in political action.

I next connect intensity theory to two empirical regularities – non-majoritarian policy and antithetical democratic citizenship – for which this model provides alternative explanations than existing political science. Proposition 4 suggests that these two phenomena, under some assumptions, are utilitarian welfare-enhancing.

9.1 Non-majoritarian policy

One key result from adding intensity of preference to a theory of electoral competition is that in equilibrium candidates sometimes propose policy with a minority of the electorate when motivated only to win votes *even when they know the majority preference*. Non-majoritarian policy outcomes have long attracted the attention of empirical political science, with many recent efforts aiming to document the extent to which implemented policy fails to reflect the preferences of citizen majorities (e.g., Lax and Phillips, 2012). This disconnect between what the majority wants and what their representatives enact is often viewed as a failure of representation.

The existence of non-majoritarian policy is so widely accepted as a stylized fact in political science that recent work has moved on to measure the mechanisms of "failure" of representation. Explanations include voters lacking the aptitude or knowledge to enact accountability (e.g., Achen and Bartels, 2016), campaign finance swaying policy in favor of the wealthy and corporations (e.g., Gilens, 2012), or politicians and their staff dramatically misunderstanding views of constituents easily measured by academic opinion surveys (e.g., Hertel-Fernandez, Mildenberger, and Stokes, 2019). While each of these stories is plausible, the first two require voters failing to act in their own interest, and the third requires candidates failing to act in theirs.

Intensity theory explains why policy is sometimes set with the minority in a simple setting with able voters and information-seeking candidates engaged in pursuit of interests. Candidates know what the majority wants with certainty and choose actions in pursuit of votes. In the model, candidates gain no benefit from campaign donations and use policy solely in pursuit of votes. Voters know what they want and instrumentally choose actions in pursuit of desired policy. Non-majority policy is nonetheless sometimes proposed because candidates are uncertain about the election outcome and so propose policy in pursuit of the votes of high-intensity voters. The theory also suggests *when* policy is proposed with the minority, which depends on the magnitude of intensity, the size of the minority, ex ante beliefs about how likely any individual voter is to be high-intensity, and what types of voters use action to communicate intensity.

The theory also suggests why electoral majorities might try to impose what appears to be conformity in behavior. If the electorate were able to *choose* whether or not to allow costly political actions (this is outside of the model) the majority would be opposed because they are strictly worse off. When there are no opportunities for the minority to differentiate themselves through costly action, policy is set with the majority with probability one. It might be worthwhile to explore what institutional arrangements might be implemented by majorities to prevent opportunities for minorities to communicate. This could explain why actors who believe they are in the majority want to lessen costs to political participation – e.g., registration requirements and other election administration rules – while actors who believe they are in the minority want to increase costs – e.g., voter identification requirements. Perhaps this would be one way to interpret efforts such as France's 2010 "Act prohibiting concealment of the face in public space."[11]

9.2 Antithetical democratic citizenship and survey responses

Students of politics have often wondered why voters seem to fall so far from the ideal democratic citizen. An ideal democratic citizen is one who comes to independent political

[11] See https://www.legifrance.gouv.fr/affichTexte.do?cidTexte=JORFTEXT000022911670.

decisions through deliberation following careful and reasoned engagement with political facts. Political scientists, on the other hand, find evidence of voters who maintain apparently biased views of political facts, express unfounded negative stereotypes about political others, endorse conspiracy theories, evince little evidence of intellectual engagement with policy debates, and support political candidates with moral deficiencies or histories of corruption.

These empirical observations lead many to conclude average voters fail as ideal democratic citizens. Intensity theory offers an alternative lens through which to view these empirical phenomena. Voters might publicly support candidates who engage in socially unacceptable behavior, support candidates of unseemly backgrounds or with histories of corruption (for a review of corruption and elections, see De Vries and Solaz, 2017), respond to demagogues, or endorse political conspiracy theories in service of communication of intensity. In the context of opinion surveys, if voters view survey responses as costly – see Bullock et al. (2015), Prior, Sood, and Khanna (2015) – obtuse answers might be efforts to communicate intensity. Respondents might give biased answers about matters of fact or claim great negative affect about political others, show favoritism towards a political group in learning political information, or say they are fine with behavior inconsistent with democratic norms. Benefits to communication provide incentives for high-intensity voters to sometimes take apparently-perverse costly action and expression.

10 Conclusion

This essay has focused on taking heterogeneity in intensity as a central part of the interaction between citizens and candidates for office. Intensity theory offers a specific interpretation of costly political action. Instead of arising out of intrinsic motivations such as duty, norms, and identities, costly action and expression allow individuals who care deeply about political issues to distinguish themselves from those who care modestly. If individuals vary in intensity, so too does incidence of costly political action and expression.

The analysis offers two key results. First, in equilibrium high-intensity voters choose costly political action in pursuit of policy goals. In the minority-only equilibrium of Proposition 1, only intense minority voters take costly action. In the separating and asymmetric equilibria (Propositions 2 and 3), however, even on an issue with a known majority, high-intensity majority voters choose costly political actions to maintain implementation of majority policy.

Second, in each equilibrium candidates propose the policy favored by the minority when candidates believe intensity of the minority is sufficiently greater than the intensity of the majority. Costly political actions allow the minority to sometimes gain representation when they care more deeply about policy. Thus, observation of failure of issue congruence – when policy is not implemented with the majority – should not necessarily lead to conclusion of failure of political representation. The focus on heterogeneity in intensity highlights a potential advantage of representative democracy over direct democracy: more utilitarian policy when minorities feel more strongly on the issue than the majority.

Proposition 4 shows that when intensity is hard to communicate, a system with costly political action can improve electorate welfare relative to a system without. This suggests that prevalence of political action and expression across many times and societies might be because citizens are better off with action. In this setting, the VCG mechanism for demand revelation from mechanism design does not improve welfare less transfers over electoral competition.

The theory extends recent work arguing that survey respondents engage in cheer-leading when asked to report beliefs about matters of fact relevant to politics (Bullock et al., 2015). A limitation of that article is it speaks to factual beliefs but not actions or attitudes. Intensity theory provides a means to evaluate incidence of strategic response to both factual and opinion questions. The meaning of survey responses depends on how costly and how public those citizens who give them believe their responses to be. If citizens believe survey responses costly and observed by candidates, survey responses might be communicating something other than unmediated reply to the question posed.

Instead of policy one might think of voters having intensity about expression of political or social identities and of candidates responding to beliefs about intensity with rhetorical expressions or symbolic actions. This model might be a useful contribution to theoretical conversations about the importance of political identities.

Political science remains uncertain about what exactly motivates individuals to volunteer for campaigns, make donations to candidates or interest groups, or participate in nomination contests. Intensity theory suggests we consider each behavioral choice part of a strategy to communicate intensity. The stronger the intensity, the more costs an individual is willing to incur. This suggests that variation in participation is driven by variation in intensity.

Because the results depend upon a simple theory and a stylized model, it is worth revisiting assumptions. I assume citizens vary in their intensity for policy and yet are not able to communicate that intensity without costly political action. Variation in preferences is almost certain to hold in large societies with heterogeneous economic, social, and cultural conditions. The second component is perhaps more in question. Intensity theory rests on an assumption that those with weak preferences do not have an incentive to accurately reveal their intensity when doing so is not too costly. Note that if majority voters are low-intensity they benefit from communication frictions because absent other information about intensity, candidates propose policy with the majority. This lends some logical support to the assumption that intensity is hard to observe. See also Dahl (1956, 99-100).

I model one election with communication during that single contest. A direction for future work is to consider multiple elections. Repeated stages, perhaps with candidates incurring costs to observe voter actions, might explain why citizen political action and expression and candidate issue positions are serially correlated across time.

If there is important variation in intensity that is hard to observe, it is useful to speculate if intensity is harder to observe on some issues than others. It might be, for example, intensity is easier to observe in the realm of economic policy because there are more opportunities for officials to observe elasticities to income. Social policy and moral issues seem more likely settings where opportunities to observe intensity are limited. It would be worth investigating if social policy is related to more strident rhetoric or more costly action than economic.

Finally, intensity theory suggests need for more careful measurement of intensity of political preference. Most survey work on policy preferences considers only the individual's ideal policy or, at best, some reduced form representation of the full utility curve across policy alternatives. The discipline would benefit from better measures. However, if one takes seriously the argument presented above and one believes respondents view opinion surveys as costly and observable, respondents do not always have incentive to accurately report intensity. This suggests need for new observational work on costly political action and expression and candidate policy responses. Intensity theory shows that care must be taken in empirical observation of political action and expression and in interpretation of citizen and candidate motives.

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Appendix

A Proof to Lemma 1: Candidate best responses

Proof. Begin by specifying expected votes for the two candidates. Voter utility from policy s is $u_i(s) = \tau_i\beta_i s + (1 - \tau_i)\beta_i(1 - s)$. The probability voter i chooses A over B given election shock δ_i is

$$
\begin{aligned}
\Pr(A) &= \Pr[\tau_i\beta_i s_A + (1 - \tau_i)\beta_i(1 - s_A) > \tau_i\beta_i s_B + (1 - \tau_i)\beta_i(1 - s_B) + \delta_i],\\
\Pr(A) &= \Pr[\beta_i s_A > \beta_i s_B + \delta_i] = \Pr[\beta_i(s_A - s_B) > \delta_i],\ i \in \{1, 2\},\\
\Pr(A) &= \Pr[\beta_i(1 - s_A) > \beta_i(1 - s_B) + \delta_i] = \Pr[\beta_i(s_B - s_A) > \delta_i],\ i = 3.
\end{aligned}
$$

Let the candidates' estimates of the intensity of Voters 1, 2, and 3 after observing $\{\lambda_1, \lambda_2, \lambda_3\}$ be $\{\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3\}$. Given the voters' weakly dominant strategy to vote for the candidate with greater expected utility and the uniform distribution on δ_i, the probability that each Voter chooses Candidate A is

$$\pi_i^A = \frac{\hat{\beta}_i(s_A - s_B) - c}{d - c},\ i \in \{1, 2\};\quad \pi_i^A = \frac{\hat{\beta}_i(s_B - s_A) - c}{d - c},\ i = 3,$$

per the cumulative distribution function for the uniform distribution with upper and lower bounds d and c (see Appendix Eq. A2).

Because the δ_i are drawn independently and the expected value of a sum of independent random variables is the sum of expectations, Candidate A's expected vote count is the sum of the three voter probabilities:

$$
\begin{aligned}
V^A &= \Big(\underbrace{\frac{\hat{\beta}_1(s_A - s_B) - c}{d - c}}_{\text{Voter 1}} + \underbrace{\frac{\hat{\beta}_2(s_A - s_B) - c}{d - c}}_{\text{Voter 2}} + \underbrace{\frac{\hat{\beta}_3(s_B - s_A) - c}{d - c}}_{\text{Voter 3}}\Big),\\
&= \big([\hat{\beta}_1 + \hat{\beta}_2][s_A - s_B] + \hat{\beta}_3[s_B - s_A]\big)/(d - c) - 3c/(d - c), \qquad \text{(A1)}
\end{aligned}
$$

with $V^B = 3 - V^A$.

Candidate A's best response to $s_B = 0$ is $s_A = 0$ when

$$
\begin{aligned}
V^A(0|s_B = 0) &\geq V^A(1|s_B = 0),\\
0 - 3c/(d - c) &\geq (\hat{\beta}_1 + \hat{\beta}_2 - \hat{\beta}_3)/(d - c) - 3c/(d - c) \Rightarrow \hat{\beta}_3 \geq \hat{\beta}_1 + \hat{\beta}_2.
\end{aligned}
$$

Candidate A's best response to $s_B = 1$ is $s_A = 0$ when

$$
\begin{aligned}
V^A(0|s_B = 1) &\geq V^A(1|s_B = 1),\\
(-\hat{\beta}_1 - \hat{\beta}_2 + \hat{\beta}_3)/(d - c) - 3c/(d - c) &\geq -3c/(d - c) \Rightarrow \hat{\beta}_3 \geq \hat{\beta}_1 + \hat{\beta}_2.
\end{aligned}
$$

Likewise, Candidate B's best response to $s_A = 0$ is $s_B = 0$ when

$$
\begin{aligned}
V^B(0|s_A = 0) &\geq V^B(1|s_A = 0),\\
3 + 3c/(d - c) &\geq 3 - (-\hat{\beta}_1 - \hat{\beta}_2 + \hat{\beta}_3)/(d - c) + 3c/(d - c),\\
0 &\geq (\hat{\beta}_1 + \hat{\beta}_2 - \hat{\beta}_3)/(d - c) \Rightarrow \hat{\beta}_3 \geq \hat{\beta}_1 + \hat{\beta}_2.
\end{aligned}
$$

Candidate B's best response to $s_A = 1$ is $s_B = 0$ when

$$\begin{aligned} V^B(0|s_A = 1) &\geq V^B(1|s_A = 1), \\ 3 - (\hat{\beta}_1 + \hat{\beta}_2 - \hat{\beta}_3)/(d - c) + 3c/(d - c) &\geq 3 + 3c/(d - c), \\ (-\hat{\beta}_1 - \hat{\beta}_2 + \hat{\beta}_3)/(d - c) &\geq 0 \Rightarrow \hat{\beta}_3 \geq \hat{\beta}_1 + \hat{\beta}_2. \end{aligned}$$

Therefore, the best response for both candidates is to propose the policy preferred by minority Voter 3, $s_A^* = s_B^* = 0$, if and only if $\hat{\beta}_1 + \hat{\beta}_2 \leq \hat{\beta}_3$. □

B Lemma A1: Candidate best responses when $-\bar{\beta} < c < -2$ and $2 < d < \bar{\beta}$

I show here that the candidate best responses described by Lemma 1 are also best responses when $-\bar{\beta} < c < -2$ and $2 < d < \bar{\beta}$.

The substantive meaning of moving $\bar{\beta}$ outside of the support of δ is that the votes of high-intensity types are deterministic rather than stochastic if the two candidates propose different policies, e.g., when $s_A = 1$ and $s_B = 0$. When $\bar{\beta}$ is larger, or $-\bar{\beta}$ smaller, than any value that might be drawn from the δ distribution, a high-intensity voter prefers a candidate who proposes their policy to one who does not with probability one.

Excluding the values $\bar{\beta}$ and $-\bar{\beta}$ from the support of election shock δ, $[c, d]$, leads to Lemma A1.

Lemma A1 (Candidate best responses when $-\bar{\beta} < c < -2$ and $2 < d < \bar{\beta}$). *When the support of election shock δ, $[c, d]$, excludes the values $\bar{\beta}$ and $-\bar{\beta}$ and $-\bar{\beta} < c < -2$ and $2 < d < \bar{\beta}$, the best response to beliefs $\{\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3\}$ for both candidates is to propose the policy preferred by minority Voter 3, $s^* = 0$, when $\hat{\beta}_1 + \hat{\beta}_2 \leq \hat{\beta}_3$, otherwise to propose the policy preferred by majority Voters 1 and 2, $s^* = 1$.*

Proof. Changing the values of c and d requires an expansion to the vote probabilities described in Lemma 1 for cases when $-\beta_i < c$ or $d < \beta_i$. The cumulative uniform distribution with upper and lower bounds b and a to value x is

$$F(x) = \begin{cases} 0, & \text{for } x < a \\ \frac{x-a}{b-a}, & \text{for } a \leq x \leq b \\ 1, & \text{for } x > b. \end{cases} \tag{A2}$$

Following Eq. A2, vote probabilities are

$$\Pr(A|\beta_i) = \begin{cases} 0, & \text{for } \beta_i(s_A - s_B) < c \\ \frac{\beta_i(s_A - s_B) - c}{d-c}, & \text{for } c \leq \beta_i(s_A - s_B) \leq d \\ 1, & \text{for } \beta_i(s_A - s_B) > d, \end{cases} \tag{A3}$$

for Voters 1 and 2, and,

$$\Pr_3(A|\beta_i) = \begin{cases} 0, & \text{for } \beta_i(s_B - s_A) < c \\ \frac{\beta_i(s_B - s_A) - c}{d-c}, & \text{for } c \leq \beta_i(s_B - s_A) \leq d \\ 1, & \text{for } \beta_i(s_B - s_A) > d, \end{cases} \tag{A4}$$

for Voter 3.

Because the δ_i are drawn independently and the expected value of a sum of independent random variables is the sum of expectations, Candidate A's expected vote count is the sum of the three voters' independent probabilities. Letting the candidates' estimates of the intensity of Voters 1, 2, and 3 after observing $\{\lambda_1, \lambda_2, \lambda_3\}$ be $\{\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3\}$, A's expected vote count is

$$V^A = \underbrace{\Pr(A|\hat{\beta}_1)}_{\text{Voter 1}} + \underbrace{\Pr(A|\hat{\beta}_2)}_{\text{Voter 2}} + \underbrace{\Pr_3(A|\hat{\beta}_3)}_{\text{Voter 3}},$$

with $V^B = 3 - V^A$.

Consider the case where $\hat{\beta}_1 + \hat{\beta}_2 \leq \hat{\beta}_3$, i.e., both majority votes are low-intensity and the minority voter high-intensity, $\beta_1 = \beta_2 = 1$ and $\beta_3 = \bar{\beta}$. Candidate A's best response to $s_B = 0$ is $s_A = 0$ when

$$\begin{aligned}
&V^A(0|s_B = 0) \geq V^A(1|s_B = 0),\\
&[\frac{0-c}{d-c} + \frac{0-c}{d-c} + \frac{0-c}{d-c}] \geq [\frac{\beta_1(1-0)-c}{d-c} + \frac{\beta_2(1-0)-c}{d-c} + 0],\\
&-3c \geq (1 - c + 1 - c - 0),\\
&-3c \geq 2 - 2c,\\
&0 \geq 2 + c \Rightarrow -\bar{\beta} \leq c \leq -2,
\end{aligned}$$

which holds by definition. A's best response to $s_B = 0$ is $s_A = 0$.

Likewise, Candidate B's best response to $s_A = 0$ is $s_B = 0$ when

$$\begin{aligned}
&V^B(0|s_A = 0) \geq V^B(1|s_A = 0),\\
&3 - [\frac{0-c}{d-c} + \frac{0-c}{d-c} + \frac{0-c}{d-c}] \geq 3 - [\frac{\beta_1(0-1)-c}{d-c} + \frac{\beta_2(0-1)-c}{d-c} + 1],\\
&3 - [0 - c + 0 - c + 0 - c]/(d-c) \geq 3 - [-1 - c - 1 - c]/(d-c) - 1,\\
&1 - 2(1+c)/(d-c) \geq -3c/(d-c),\\
&d - c - 2 - 2c \geq -3c,\\
&d \geq 2 \Rightarrow 2 \leq d \leq \bar{\beta},
\end{aligned}$$

which holds by definition. B's best response to $s_A = 0$ is $s_B = 0$.

Therefore, proposing policy with the minority $s_A^* = s_B^* = 0$ is a mutual best response when $\hat{\beta}_1 + \hat{\beta}_2 \leq \hat{\beta}_3$ and $-\bar{\beta} < c < -2$ and $2 < d < \bar{\beta}$. □

Combining Lemma 1 and Lemma A1 provides the requirement for non-majoritarian policy that $c < -2$ and $d > 2$. Lemma 1 shows that $s^* = 0$ if and only if $\hat{\beta}_1 + \hat{\beta}_2 \leq \hat{\beta}_3$, which can only occur if $\bar{\beta} > 2$. As the Lemma assumes $c < -\bar{\beta}$ and $\bar{\beta} < d$, c must be less than -2 and d greater than 2. The proof to Lemma A1 also shows that the equilibrium requires $c < -2$ and $d > 2$.

C Proof to Proposition 1: Minority policy representation through political action

When $\bar{\beta} < 2$, equilibrium policy is $s^* = 1$ per Lemma 1 because always $\beta_1 + \beta_2 > \beta_3$. We are interested in a setting where candidates might propose policy with the minority and so I make the assumption $\bar{\beta} > 2$ for the remainder of this proof.

A strategy for a voter is a function $\sigma_v(\beta_i) : \{1, \bar{\beta}\} \to \mathbb{R}_+$ mapping intensity into political action $\lambda_i \geq 0$. Beliefs for a candidate, $p \in \{A, B\}$, are a function $h_p(\lambda_1, \lambda_2, \lambda_3) : \mathbb{R}^3_+ \to \Delta(\{1, \bar{\beta}\}^3)$ mapping observed political action into lotteries over voter intensity types. A strategy for a candidate, $p \in \{A, B\}$, is a function $\sigma_p(\lambda_1, \lambda_2, \lambda_3) : \mathbb{R}^3_+ \to \{0, 1\}$ mapping observed political action into policy platform s_p.

Consider an equilibrium magnitude of political action, λ^*, satisfying

$$\begin{aligned} &1 \leq \lambda^* \leq \bar{\beta}, \\ &q \in (0, 1 - \sqrt{2}/2). \end{aligned} \tag{A5}$$

Claim: there is a perfect Bayesian equilibrium $(\sigma^*_1, \sigma^*_2, \sigma^*_3, \sigma^*_A, \sigma^*_B, h^*_A, h^*_B)$ in which

$$\begin{aligned} &h^*_p(\lambda_1) \to \Pr(\beta_1 = \bar{\beta}) = q,\ \Pr(\beta_1 = 1) = 1 - q,\ p \in \{A, B\},\ \forall \lambda_1 \in \mathbb{R}_+, \\ &h^*_p(\lambda_2) \to \Pr(\beta_2 = \bar{\beta}) = q,\ \Pr(\beta_2 = 1) = 1 - q,\ p \in \{A, B\},\ \forall \lambda_2 \in \mathbb{R}_+, \\ &h^*_p(\lambda_3 < \lambda^*) \to \beta_3 = 1,\ p \in \{A, B\}, \\ &h^*_p(\lambda_3 \geq \lambda^*) \to \beta_3 = \bar{\beta},\ p \in \{A, B\}, \\ &\sigma^*_i(1) = \sigma^*_i(\bar{\beta}) = 0,\ i \in \{1, 2\}, \\ &\sigma^*_3(1) = 0,\ \sigma^*_3(\bar{\beta}) = \lambda^*, \\ &\sigma^*_A(\lambda) = \sigma^*_B(\lambda) = 0 \iff \lambda_3 \geq \lambda^*, \\ &\text{else } \sigma^*_A(\lambda) = \sigma^*_B(\lambda) = 1. \end{aligned} \tag{A6}$$

Proof. Suppose voters and candidates follow (A6). Then, when candidates observe $\lambda_3 \geq \lambda^*$, they assign probability one that $\beta_3 = \bar{\beta}$. Candidates assign probability q that $\beta_i = \bar{\beta}$ for both Voters 1 and 2, and $1 - q$ $\beta_i = 1$.

Policy $s^* = 0$ occurs when candidates believe $\beta_1 + \beta_2 \leq \beta_3$. When candidates have assigned probability one to $\beta_3 = \bar{\beta}$, there is probability $(1 - q)^2$ that $\beta_1 + \beta_2 \leq \beta_3$ and probability $1 - (1 - q)^2$ that $\beta_1 + \beta_2 > \beta_3$. The policy equilibrium in pure strategies is $s^* = 0$ when

$$\begin{aligned} (1 - q)^2 &\geq 1 - (1 - q)^2, \\ 1 - 2q + q^2 &\geq 1 - 1 + 2q - q^2, \\ 1 - 4q + 2q^2 &\geq 0, \end{aligned}$$

which occurs when $q < 1 - \sqrt{2}/2 \approx 0.3$. Proposing policy $s^* = 0$ if and only if $\lambda_3 \geq \lambda^*$ else $s^* = 1$ is therefore optimal on the equilibrium path when $q < 1 - \sqrt{2}/2$.

Turning to the voters, begin with minority Voter 3. If $\beta_3 = 1$, then $\sigma^*_3(1) = 0$ and Voter

3's expected benefit is zero because this choice guarantees $s^* = 1$ per (A6). The expected payoff from deviating and choosing $\lambda_3 = \lambda^*$ is $1 - \lambda^*$. Given $1 \leq \lambda^*$ (Req. A5), Voter 3 does not benefit from this deviation.

Now, consider $\beta_3 = \bar{\beta}$. $\sigma_3^*(\bar{\beta}) = \lambda^*$ yields an expected benefit of $\bar{\beta} - \lambda^*$. Deviation from the strategy to $\lambda_3 = 0$ has an expected benefit of zero. The equilibrium strategy is optimal if $\bar{\beta} - \lambda^* \geq 0$, which follows from the bounds presented in (A5).

There are two majority voter types, high-intensity and low-intensity. Without loss of generality to Voter 2, consider Voter 1. When $\beta_1 = 1$, $\sigma_1^*(1) = 0$ with an expected payoff of $(1 - q)(\beta_1)$, the probability that Voter 3 is low-intensity and therefore abstains from costly action multiplied by Voter 1's payoff to policy $s = 1$, $\beta_1 = 1$. Because of candidate beliefs $h_A^* = h_B^*$, deviating and choosing $\lambda_1 = \lambda^*$ has an expected payoff of $(1 - q)(\beta_1) - \lambda^*$. Because $\lambda^* \geq 0$, the low-intensity majority voter does not benefit from deviating.

Finally, when $\beta_1 = \bar{\beta}$, $\sigma_1^*(\bar{\beta}) = 0$ has an expected payoff of $(1 - q)\bar{\beta}$. Because of candidate beliefs $h_A^* = h_B^*$, deviating and choosing $\lambda_1 = \lambda^*$ has an expected payoff of $(1 - q)(\bar{\beta}) - \lambda^*$. Because $\lambda^* \geq 0$, the high-intensity majority voter does not benefit from deviating. Majority voters do not deviate as long as $0 \leq \lambda^*$.

A *Minority policy representation through political action* equilibrium holds with the strategies and beliefs presented in Req. A6 when $q < 1 - \sqrt{2}/2$ and when λ^* follows the bounds $1 \leq \lambda^* \leq \bar{\beta}$. □

Applying an intuitive refinement generates a unique equilibrium where λ^* equals its lower bound of one. Suppose the candidates observe an off-the-path deviation where $\lambda_3 < 1$ and assume that magnitude of costly action indicates high-intensity. Would low-intensity voters deviate from the equilibrium $\sigma_i^*(1) = 0$. They would deviate because when the candidates believe either of the majority voters high-intensity, they set policy with the majority. As $0 < \lambda < \beta = 1$, the majority voters would deviate.

D Proof to Proposition 2: Representation through separating political action

When $\bar{\beta} < 2$, equilibrium policy is $s^* = 1$ per Lemma 1 because always $\beta_1 + \beta_2 > \beta_3$. We are interested in a setting where candidates might propose policy with the minority and so I make the assumption $\bar{\beta} > 2$ for the remainder of this proof.

A strategy for a voter is a function $\sigma_v(\beta_i) : \{1, \bar{\beta}\} \rightarrow \mathbb{R}_+$ mapping intensity into political action $\lambda_i \geq 0$. Beliefs for a candidate, $p \in \{A, B\}$, are a function $h_p(\lambda_1, \lambda_2, \lambda_3) : \mathbb{R}^3_+ \rightarrow \Delta(\{1, \bar{\beta}\}^3)$ mapping observed political action into lotteries over voter intensity types. A strategy for a candidate, $p \in \{A, B\}$, is a function $\sigma_p(\lambda_1, \lambda_2, \lambda_3) : \mathbb{R}^3_+ \rightarrow \{0, 1\}$ mapping observed political action into policy platform s_p.

Consider an equilibrium magnitude of political action, λ^*, satisfying

$$\begin{aligned}
(1-q)^2 \leq \lambda^* \leq \bar{\beta}(1-q-(1-q)^2) & \text{ if } q < 1/2, \\
1-q-(1-q)^2 \leq \lambda^* \leq \bar{\beta}(1-q)^2 & \text{ if } q \geq 1/2, \\
q \in (1/(\bar{\beta}+1),\ \bar{\beta}/(\bar{\beta}+1)). &
\end{aligned} \tag{A7}$$

Claim: there is a perfect Bayesian equilibrium $(\sigma_1^*, \sigma_2^*, \sigma_3^*, \sigma_A^*, \sigma_B^*, h_A^*, h_B^*)$ in which

$$\begin{aligned}
& h_p^*(\lambda_i < \lambda^*) \rightarrow \beta_i = 1,\ i \in \{1,2,3\},\ p \in \{A, B\} \\
& h_p^*(\lambda_i \geq \lambda^*) \rightarrow \beta_i = \bar{\beta},\ i \in \{1,2,3\},\ p \in \{A, B\} \\
& \sigma_i^*(1) = 0,\ \sigma_i^*(\bar{\beta}) = \lambda^*,\ i \in \{1,2,3\}, \\
& \sigma_A^*(\lambda) = \sigma_B^*(\lambda) = 1 \text{ if} \\
& \quad 1[\lambda_1 \geq \lambda^*] + 1[\lambda_2 \geq \lambda^*] \geq 1[\lambda_3 \geq \lambda^*] \\
& \text{and } \sigma_A^*(\lambda) = \sigma_B^*(\lambda) = 0 \text{ otherwise,}
\end{aligned} \tag{A8}$$

where $1[\cdot]$ is the indicator function.

Proof. Suppose voters and candidates follow (A8). Then, candidates believe $\beta_1 + \beta_2 \geq \beta_3$ with probability one when observing λ such that $1[\lambda_1 \geq \lambda^*] + 1[\lambda_2 \geq \lambda^*] \geq 1[\lambda_3 \geq \lambda^*]$ and form the belief that $\beta_1 + \beta_2 \geq \beta_3$ with probability zero when λ is otherwise. Given these beliefs and Lemma 1, the candidate strategy (A8) is optimal on the equilibrium path.

Turning to the voters, begin with minority Voter 3. If $\beta_3 = 1$, then $\sigma_3^*(1) = 0$ and Voter 3's expected benefit is 0 because this choice guarantees $1[\lambda_1 \geq \lambda^*] + 1[\lambda_2 \geq \lambda^*] \geq 1[\lambda_3 \geq \lambda^*]$ and $s^* = 1$. The expected payoff from deviating and choosing $\lambda_3 = \lambda^*$ is the probability that neither majority voter chooses λ^* times the benefit of policy $s = 0$ less the cost λ^*, or $(1-q)^2(\beta_3) - \lambda^*$. Given $\beta_3 = 1$ and $(1-q)^2 \leq \lambda^*$ (Req. A7), Voter 3 does not benefit from this deviation.

Now, consider $\beta_3 = \bar{\beta}$. $\sigma_3^*(\bar{\beta}) = \lambda^*$ yields an expected benefit of $\bar{\beta}(1-q)^2 - \lambda^*$. Deviation from the strategy to $\lambda_3 = 0$ has an expected benefit of zero as it guarantees the candidates choose $s = 1$. The equilibrium strategy is optimal if $\bar{\beta}(1-q)^2 - \lambda^* \geq 0$, which follows from the bounds presented in (A7).

Majority Voters 1 and 2 have the same payoffs, beliefs, and actions, so the proof for Voter 1 is the same as for 2. The postulated strategy when $\beta_1 = 1$ is $\sigma_1^*(1) = 0$ with an expected payoff of $(q + (1 - q)^2)(\beta_1)$, the probability that Voter 2 is high-intensity and chooses costly action plus the probability that both Voter 2 and Voter 3 are low-intensity, each multiplied by Voter 1's payoff to policy s = 1, $\beta_1 = 1$. Deviating and choosing $\lambda_1 = \lambda^*$ has an expected payoff of $\beta_1 - \lambda^*$ because whenever either majority voter chooses costly action, policy is set at the majority preference. Voter 1 does not deviate if $1-\lambda^* \leq q+(1-q)^2$, which holds by the bounds in (A7).

Finally, if $\beta_1 = \bar{\beta}$, then following the postulated strategy $\sigma_1^*(\bar{\beta}) = \lambda^*$ has an expected payoff of $\bar{\beta} - \lambda^*$. Deviating with $\lambda_1 = 0$ yields $\bar{\beta}(q + (1 - q)^2)$ and Voter 1 does not deviate if $\bar{\beta}(q + (1 - q)^2) \leq \bar{\beta} - \lambda^*$, which follows the bounds in (A7).

A separating equilibrium holds with the strategies and beliefs presented in Req. A8 and when λ^* follows the bounds $\max\{(1 - q)^2, 1 - q - (1 - q)^2\} \leq \lambda^* \leq \min\{\bar{\beta}(1 - q)^2, \bar{\beta}(1 - q - (1 - q)^2)\}$. The equilibrium lower bound on λ^* follows the lower bound of minority Voter 3 $(1 - q)^2$ when

$$\begin{aligned}(1 - q)^2 &> 1 - q - (1 - q)^2,\\ 1 - 2q + q^2 &> 1 - q - 1 + 2q - q^2,\\ 1 &> 3q - 2q^2 \rightarrow (1 - 2q)(1 - q) > 0,\end{aligned}$$

which obtains if and only if $q < 1/2$. It follows, then, that the equilibrium holds when λ^* is within the bounds

$$\begin{aligned}(1 - q)^2 \leq \lambda^* &\leq \bar{\beta}(1 - q - (1 - q)^2) \text{ if } q < 1/2,\\ 1 - q - (1 - q)^2 \leq \lambda^* &\leq \bar{\beta}(1 - q)^2 \text{ if } q \geq 1/2.\end{aligned}$$

Finally, q must be consistent with the bounds in (A7)

$$\begin{aligned}\text{If } q < 1/2 :\ & (1 - q)^2 \leq \bar{\beta}(1 - q - (1 - q)^2)\\ & (1 - q)^2/(q - q^2) \leq \bar{\beta}\\ & (1 - q)/q \leq \bar{\beta} \quad \rightarrow \quad 1/(1 + \bar{\beta}) \leq q.\\ \text{If } q \geq 1/2 :\ & 1 - q - (1 - q)^2 \leq \bar{\beta}(1 - q)^2\\ & q - q^2 \leq \bar{\beta}(1 - q)^2, \quad q \leq (1 - q)\bar{\beta},\\ & q(1 + \bar{\beta}) \leq \bar{\beta} \rightarrow q \leq \bar{\beta}/(\bar{\beta} + 1).\end{aligned}$$

A *Representation through separating political action* equilibrium holds with the strategies and beliefs presented in Req. A8 when $q \in (1/(\bar{\beta} + 1), \bar{\beta}/(\bar{\beta} + 1))$ and when λ^* follows the bounds in Req. A7. □

Applying an intuitive refinement generates a unique equilibrium where λ^* equals $(1-q)^2$ when $q < 1/2$ and $1-q-(1-q)^2$ when $q \geq 1/2$. Suppose the candidates observe an off-the-path deviation where λ_i is lower than these values and candidates assume that magnitude of costly action indicates high-intensity. By the proof above, low-intensity voters would deviate because they can profit by choosing a magnitude of costly action that in expectation gains them greater policy benefits than costs. Only costs at the lower bounds described above satisfy the intuitive criterion.

E Proof to Proposition 3: Representation through asymmetric majority political action

For this asymmetric equilibrium, I will assume Voter 1 is the never-action actor and Voter 2 chooses costly action when high-intensity. However, an equivalent equilibrium would hold with Voter 2 never-action and Voter 1 separating.

When $\bar{\beta} < 2$, equilibrium policy is $s^* = 1$ per Lemma 1 because always $\beta_1 + \beta_2 > \beta_3$. We are interested in a setting where candidates might propose policy with the minority and so I make the assumption $\bar{\beta} > 2$ for the remainder of this proof.

A strategy for a voter is a function $\sigma_v(\beta_i) : \{1, \bar{\beta}\} \to \mathbb{R}_+$ mapping intensity into political action $\lambda_i \geq 0$. Beliefs for a candidate, $p \in \{A, B\}$, are a function $h_p(\lambda_1, \lambda_2, \lambda_3) : \mathbb{R}^3_+ \to \Delta(\{1, \bar{\beta}\}^3)$ mapping observed political action into lotteries over voter intensity types. A strategy for a candidate, $p \in \{A, B\}$, is a function $\sigma_p(\lambda_1, \lambda_2, \lambda_3) : \mathbb{R}^3_+ \to \{0, 1\}$ mapping observed political action into policy platform s_p.

Consider an equilibrium magnitude of political action, λ^*, satisfying

$$\begin{aligned} & q \leq \lambda^* \leq (1 - q)\bar{\beta}, \\ & 1/2 < q < \bar{\beta}/(\bar{\beta} + 1). \end{aligned} \tag{A9}$$

Claim: there is a perfect Bayesian equilibrium $(\sigma_1^*, \sigma_2^*, \sigma_3^*, \sigma_A^*, \sigma_B^*, h_A^*, h_B^*)$ in which

$$\begin{aligned} & h_p^*(\lambda_1) \to \Pr(\beta_1 = \bar{\beta}) = q,\ \Pr(\beta_1 = 1) = 1 - q,\ p \in \{A, B\},\ \forall \lambda_1 \in \mathbb{R}_+, \\ & h_p^*(\lambda_i < \lambda^*) \to \beta_i = 1,\ i \in \{2, 3\},\ p \in \{A, B\} \\ & h_p^*(\lambda_i \geq \lambda^*) \to \beta_i = \bar{\beta},\ i \in \{2, 3\},\ p \in \{A, B\} \\ & \sigma_1^*(1) = \sigma_1^*(\bar{\beta}) = 0, \\ & \sigma_i^*(1) = 0,\ \sigma_i^*(\bar{\beta}) = \lambda^*,\ i \in \{2, 3\}, \\ & \sigma_A^*(\lambda) = \sigma_B^*(\lambda) = 0 \iff \lambda_3 \geq \lambda^*, \lambda_2 < \lambda^* \\ & \text{and } \sigma_A^*(\lambda) = \sigma_B^*(\lambda) = 1 \text{ otherwise.} \end{aligned} \tag{A10}$$

Proof. Suppose voters and candidates follow (A10). Then, when candidates observe $\lambda_2 \geq \lambda^*$ they assign probability one that $\beta_2 = \bar{\beta}$ and when $\lambda_3 \geq \lambda^*$ they assign probability one that $\beta_3 = \bar{\beta}$. Candidates assign probability q that $\beta_1 = \bar{\beta}$ and $1 - q$ $\beta_1 = 1$.

Policy $s^* = 0$ occurs when candidates believe $\beta_1 + \beta_2 \leq \beta_3$. When candidates have assigned probability zero to $\beta_2 = \bar{\beta}$ and probability one to $\beta_3 = \bar{\beta}$, there is probability $(1 - q)$ that $\beta_1 + \beta_2 \leq \beta_3$ and probability $1 - (1 - q)$ that $\beta_1 + \beta_2 > \beta_3$. The policy equilibrium in pure strategies is $s^* = 0$ when

$$\begin{aligned} & (1 - q) \geq 1 - (1 - q), \\ & 1 - 2q \geq 0, \end{aligned}$$

which occurs when $q > 1/2$. Proposing policy $s^* = 0$ if and only if $\lambda_3 \geq \lambda^*$ and $\lambda_2 < \lambda^*$ else $s^* = 1$ is therefore optimal on the equilibrium path when $q > 1/2$.

Turning to the voters, begin with minority Voter 3. If $\beta_3 = 1$, then $\sigma_3^*(1) = 0$ and Voter 3's expected benefit is zero because this choice guarantees $s^* = 1$ per (A10). The expected payoff from deviating and choosing $\lambda_3 = \lambda^*$ is (1 – q)(β_3) – λ^*, which is the probability that Voter 2 is low-intensity and abstains from costly action times the benefit to Voter 3 of obtaining policy. Given 1 – q $\leq \lambda^*$ and q $>$ 1/2 (Req. A9), Voter 3 does not benefit from this deviation.

Now, consider $\beta_3 = \bar{\beta}$. $\sigma_3^*(\bar{\beta}) = \lambda^*$ yields an expected benefit of (1 – q)$\bar{\beta}$ – λ^*. Deviation from the strategy to λ_3 = 0 has an expected benefit of zero. The equilibrium strategy is optimal if (1 – q)$\bar{\beta}$ – $\lambda^* \geq 0$, which follows from the bounds presented in (A9).

Continuing with Voter 2, if β_2 = 1, then $\sigma_2^*(1)$ = 0 and Voter 2's expected benefit is (1 – q)(β_2), the probability that Voter 3 is low-intensity and abstains from costly action times the benefit to Voter 2 of obtaining policy. The expected payoff from deviating and choosing $\lambda_2 = \lambda^*$ is (β_2) – λ^*. Voter 2 does not benefit from deviating if 1 – q $>$ 1 – λ^*. Given q $\leq \lambda^*$ (Req. A9), Voter 2 does not benefit from this deviation.

When $\beta_2 = \bar{\beta}$, $\sigma_2^*(\bar{\beta}) = \lambda^*$ and Voter 2's expected benefit is $\bar{\beta}$ – λ^*. The expected payoff from deviating and choosing λ_2 = 0 is (1 – q)$\bar{\beta}$, the probability that Voter 3 is low-intensity and abstains from costly action times the benefit to Voter 2 of obtaining policy. Voter 2 does not benefit from deviating if $\bar{\beta}$ – $\lambda^* >$ (1 – q)$\bar{\beta}$. Given $\lambda^* \leq$ q$\bar{\beta}$ and q $>$ 1/2 (Req. A9), Voter 2 does not benefit from this deviation.

Finally, Voter 1 abstains from costly action regardless of type. When β_3 = 1, $\sigma_3^*(1)$ = 0 with an expected payoff of (q)(β_1) + (1 – q)^2(β_1), the probability that Voter 2 is high-intensity and induces s* = 1 with costly action multiplied by Voter 1's payoff to policy s = 1, β_1 = 1 plus the probability that both Voter 2 and Voter 3 are low-intensity (1 – q)^2, both abstain from costly action, and policy is s* = 1. Because of candidate beliefs $h_A^* = h_B^*$, deviating and choosing $\lambda_1 = \lambda^*$ has an expected payoff of (q)(β_1) + (1 – q)^2(β_1) – λ^*. Because $\lambda^* \geq 0$, the low-intensity Voter 1 does not benefit from deviating.

Finally, when $\beta_1 = \bar{\beta}$, $\sigma_1^*(\bar{\beta})$ = 0 has an expected payoff of (q)$\bar{\beta}$ + (1 – q)$^2\bar{\beta}$. Because of candidate beliefs $h_A^* = h_B^*$, deviating and choosing $\lambda_1 = \lambda^*$ has an expected payoff of (1 – q)($\bar{\beta}$) – λ^*. Because $\lambda^* \geq 0$, the high-intensity majority voter does not benefit from deviating. Majority voters do not deviate as long as $0 \leq \lambda^*$.

An asymmetric majority equilibrium holds with the strategies and beliefs presented in Req. A10 and when λ^* follows the bounds $\max\{(1 - q), q\} \leq \lambda^* \leq \min\{(1 - q)\bar{\beta}, q\bar{\beta}\}$. The equilibrium lower bound on λ^* follows the lower bound of minority Voter 3 when (1–q) $>$ q, q $<$ 1/2. However, because q $>$ 1/2 is a requirement for the equilibrium, the lower bound follows that of Voter 2, q. The upper bound follows the upper bound of minority Voter 3 when (1 – q)$\bar{\beta} <$ q$\bar{\beta}$, which always holds because q $>$ 1/2. Therefore, q $\leq \lambda^* \leq$ (1 – q)$\bar{\beta}$.

Finally, q must be consistent with the bounds in (A9)

$$
\begin{aligned}
q &\leq \bar{\beta}(1 - q), \\
q(1 + \bar{\beta}) &\leq \bar{\beta}, \\
q &\leq \bar{\beta}/(\bar{\beta} + 1).
\end{aligned}
$$

A *Representation through asymmetric majority political action* equilibrium holds with the strategies and beliefs presented in Req. A10 when 1/2 $<$ q $<$ $\bar{\beta}/(\bar{\beta}+1)$ and when λ^* follows the bounds q $\leq \lambda^* \leq$ (1 – q)$\bar{\beta}$. □

Applying an intuitive refinement generates a unique equilibrium where λ^* = q. Suppose the candidates observe an off-the-path deviation where $\lambda_i < q$ and candidates assume that magnitude of costly action indicates high-intensity. By the proof above, low-intensity voters would deviate because they can profit by choosing a magnitude of costly action that in expectation gains them greater policy benefits than costs. Only at λ^* = q is intuitive.

F Two policy dimensions

In this section, I extend the model to include party competition over two dimensions. The results show that adding a second dimension of policy conflict does not change the importance of heterogeneous intensity or consequent incentives for high-intensity voters to incur costly action to communicate intensity. On the first policy dimension without intensity, however, both candidates' dominant strategy is to propose policy with the majority. This highlights how intensity is a necessary condition for non-majoritarian policy.

The model again has two vote-maximizing candidates, A and B, and an electorate of three voters. In this version, voters have preferences over two binary policies r and s. Voter policy preferences are tuples over r and s, $\{\tau_i^r \in \{0,1\},\ \tau_i^s \in \{0,1\}\}$, common knowledge as before. Without loss of generality, assume Voters 1, 2, and 3 have preferences $\{0,1\}$, $\{1,1\}$, and $\{1,0\}$ so that r = 1 and s = 1 are majority positions.

In addition to policy preference, voters vary in the *intensity* with which they care about issue s, $\beta_i \in \{1, \bar{\beta}\}$, $\bar{\beta} > 1$, representing low- and high-intensity. If s is set at the voter's preference (e.g., s = 0 for a τ_i^s-0 voter), their payoff is β_i, $\bar{\beta}$ if high-intensity or 1 if low-intensity. Payoff is zero when policy is set opposite their preference. Intensity β_i is private knowledge for each voter i. However, the ex ante rate q that voters are high-intensity is common knowledge, $\Pr(\beta_i = \bar{\beta}) = q$ and $\Pr(\beta_i = 1) = 1 - q$, $q \in (0,1)$.

Voters do not vary in intensity for policy r, gaining utility 1 when $r = \tau_i^r$, else zero. One might thus interpret $\bar{\beta}$ as how much more high-intensity types value policy on s over r.

F.1 Actions

Candidates each take one action, simultaneously proposing binding policy tuples $\{r_A, s_A\}$ and $\{r_B, s_B\}$. Voters take two actions. At the election, voters make a vote choice given $\tau_i, \beta_i, r_A, r_B, s_A$, and s_B. Vote choice is a random variable, represented in the model with independent additive election-shock δ_i. The random component is revealed at the time of the election and drawn according to the uniform distribution with upper and lower bounds c and d common knowledge. Because preference-τ type-β voter payoffs to policy are $u_i(r,s) = \tau_i^r r + (1 - \tau_i^r)(1 - r) + \tau_i^s \beta_i s + (1 - \tau_i^s)\beta_i(1 - s)$, the probability each voter chooses Candidate A over Candidate B is

$$\begin{aligned} &\Pr[r_B - r_A + \beta_i(s_A - s_B) > \delta_i] \text{ (Voter 1)}, \\ &\Pr[r_A - r_B + \beta_i(s_A - s_B) > \delta_i] \text{ (Voter 2)}, \\ &\Pr[r_A - r_B + \beta_i(s_B - s_A) > \delta_i] \text{ (Voter 3)}. \end{aligned} \tag{A11}$$

As before, the voters' second action is choice over magnitude of political actions of (net) cost $\lambda_i \in \mathbb{R}_+$.

F.2 Timing

The timing of the game is

1. Nature independently draws each β_i, $i \in 1,2,3$ from $\{1, \bar{\beta}\}$ given q.
2. Voters privately observe β_i and then simultaneously choose pre-election actions λ_i.
3. Candidates observe $(\lambda_1, \lambda_2, \lambda_3)$ then propose policy platforms $\{r_A, s_A\}$ and $\{r_B, s_B\}$.

4. Nature independently draws each δ_i and election held. Candidate with majority wins election.
5. Payoffs realized.

F.3 Strategies and beliefs

At the election, voters have a weakly dominant strategy to vote for their preferred candidate and when indifferent choose A with probability 0.5 and B with probability 0.5. Voter strategy over λ is a function $\sigma_V(\beta_i) : \{1, \bar{\beta}\} \to \mathbb{R}_+$ mapping intensity into political action λ_i. For both candidates, a strategy is a function $\sigma_p(\lambda_1, \lambda_2, \lambda_3) : \mathbb{R}^3_+ \to \{0, 1\}^2,\ p \in \{A, B\}$, mapping observed political actions into a policy platform $\{r_p,\ s_p\}$.

Candidates learn about intensity by the costly actions taken by each voter and use Bayes' Rule to update beliefs about β so as to propose platforms to maximize probability of election. Because the candidates' information and learning technologies are equivalent, so too are their beliefs. As before, PBE is the solution concept.

F.4 Lemma A2

The following lemma shows that with the second policy dimension candidate best response on s is the same as in Lemma 1: candidates propose s with high-intensity voters. With this Lemma, all subsequent results of the paper then follow. That is, even with the second dimension, costly political action arises in equilibrium and policy on the s dimension is set with the minority when high-intensity and the majority low-intensity. On the r dimension with intensity homogeneous, however, policy is always set with the majority $r^* = 1$.

Lemma A2 (Candidate best responses, two dimensions)**.** *Best response for both candidates is to propose* $r^* = 1$ *(with the majority) on policy* r, *and on policy* s *propose the policy preferred by minority Voter 3*, $s^* = 0$, *if and only if* $\hat{\beta}_1 + \hat{\beta}_2 \leq \hat{\beta}_3$, *otherwise* $s^* = 1$.

Proof. Begin by specifying the expected vote count for the two candidates. Let the candidates' estimates of the intensity of Voters 1, 2, and 3 after observing $\{\lambda_1, \lambda_2, \lambda_3\}$ be $\{\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3\}$. Given the voters' weakly dominant strategy to vote for the candidate with greater expected utility and the uniform distribution on δ, expected vote count for Candidate A is

$$
\begin{aligned}
V^A &= \frac{1 - r_A - (1 - r_B) + \hat{\beta}_1(s_A - s_B) - c}{d - c} + \frac{r_A - r_B + \hat{\beta}_2(s_A - s_B) - c}{d - c} \\
&\quad + \frac{r_A - r_B + \hat{\beta}_3(s_B - s_A) - c}{d - c}, \\
&= \big(r_A - r_B + [\hat{\beta}_1 + \hat{\beta}_2][s_A - s_B] + \hat{\beta}_3(s_B - s_A)\big)/(d - c) - 3c/(d - c),
\end{aligned}
$$

with $V^B = 3 - V^A$.

Note that vote share is additively separable in actions r and s. Because it is also linear, candidate actions on s and r are independent, and the best responses on s from Lemma 1 hold.

Consider the best responses of each candidate in selecting policy. Candidate A's best response to $r_B = 0$ is $r_A = 1$ when

$$\begin{aligned} V^A(1|r_B = 0) &\geq V^A(0|r_B = 0), \\ (1-0)/(d-c) - 3c/(d-c) &\geq -3c/(d-c) \Rightarrow 1 \geq 0. \end{aligned}$$

Candidate A's best response to $r_B = 1$ is $r_A = 1$ when

$$\begin{aligned} V^A(1|r_B = 1) &\geq V^A(0|r_B = 1), \\ (1-1)/(d-c) - 3c/(d-c) &\geq (0-1)/(d-c) - 3c/(d-c) \Rightarrow 0 \geq -1. \end{aligned}$$

Candidate B's best response to $r_A = 0$ is $r_B = 1$ when

$$\begin{aligned} V^B(1|r_A = 0) &\geq V^B(0|r_A = 0), \\ 3 - ((0-1)/(d-c) - 3c/(d-c)) &\geq 3 + 3c/(d-c), \\ 1/(d-c) + 3c/(d-c) &\geq 3c/(d-c) \Rightarrow 1 \geq 0. \end{aligned}$$

Candidate B's best response to $r_A = 1$ is $r_B = 1$ when

$$\begin{aligned} V^B(1|r_A = 1) &\geq V^B(0|r_A = 1), \\ 3 - ((1-1)/(d-c) - 3c/(d-c)) &\geq 3 - ((1-0)/(d-c) - 3c/(d-c)), \\ 3c/(d-c) &\geq -1/(d-c) + 3c/(d-c) \Rightarrow 0 \geq -1. \end{aligned}$$

For both candidates, dominant strategy is to propose $r^* = 1$. □

G Equilibrium magnitude of action

In this section I compare the magnitude of equilibrium costly political action λ^* required in the three equilibria of interest. The bounds on λ^* are

$$
\begin{aligned}
&1 \leq \lambda^* \leq \bar{\beta} \quad \text{(Prop 1)},\\
&(1-q)^2 \leq \lambda^* \leq \bar{\beta}(1-q-(1-q)^2) \quad \text{if } q < 1/2, \text{ and}\\
&1-q-(1-q)^2 \leq \lambda^* \leq \bar{\beta}(1-q)^2 \quad \text{if } q \geq 1/2, \quad \text{(Prop 2)},\\
&q \leq \lambda^* \leq \bar{\beta}/(\bar{\beta}+1) \quad \text{(Prop 3)}.
\end{aligned}
$$

Because $B(1-q)^2 < B$ and $B(1-q-(1-q)^2) < B$ for $B = 1$ and $B = \bar{\beta}$, both bounds of Proposition 2 are strictly less than the corresponding bounds of Proposition 1.

Because $q < 1$ and $\bar{\beta}/(\bar{\beta}+1) < \bar{\beta}$, both bounds of Proposition 3 are strictly less than the corresponding bounds of Proposition 1.

Equilibrium magnitude of action is lower in the separating equilibrium than in the asymmetric equilibrium if

$$
\begin{aligned}
(1-q)^2 < q \text{ and } \bar{\beta}(1-q-(1-q)^2) < (1-q)\bar{\beta} \text{ if } q < 1/2,\\
1-q-(1-q)^2 < q \text{ and } \bar{\beta}(1-q)^2 < (1-q)\bar{\beta} \text{ if } q \geq 1/2.
\end{aligned}
$$

When $q < 1/2$, the first term holds when

$$(1-q)^2 < q, \; 1-2q+q^2 < q, \; q^2-3q+1 < 0,$$

which holds when $q < (3-\sqrt{5})/2 \approx 0.38$. The second term holds when

$$(1-q-(1-q)^2) < 1-q, \; q(1-q) < 1-q, \; q < 1,$$

which holds for all q.

When $q \geq 1/2$, the first term holds when

$$1-q-(1-q)^2 < q, \; q-q^2 < q, \; -q^2 < 0,$$

which holds for all q. The second term holds when

$$(1-q)^2 < (1-q)\bar{\beta}, \; (1-q) < \bar{\beta},$$

which holds for all q.

Equilibrium magnitude of action is lower in the separating equilibrium when $q < (3-\sqrt{5})/2$ or $q \geq 1/2$.

H Welfare analysis

In this section, I analyze welfare and provide a proof for Proposition 4. I evaluate welfare under three different mechanisms to allocate policy in the setting of three voters with (common knowledge) heterogeneous preferences and (private information) heterogeneous intensity. First, a mechanism where individual intensity is not communicated to the candidates for office and candidates choose policy based on prior beliefs about the distribution of intensity (Setting 1). Second, a mechanism as in the separating equilibrium of Proposition 2 where voters might incur costly political action to communicate their private-information intensity (Setting 2). Third, a Vickrey-Clark-Groves (VCG) mechanism where voters send a message revealing their private value to policy used by a social planner to make the social choice (Setting 3).

H.1 Notation and definitions

The electoral Settings 1 and 2 are similar to that described in the main text with three voters and two office-seeking candidates. Setting 3 has not candidates but instead social choice is determined by a VCG mechanism. I use the following notation and definitions.

Voters There are three voters, Voter 1, Voter 2, and Voter 3.

Candidates There are two candidates A and B.

Decision One mutually-exclusive decision s must be made from the pair $\{0, 1\}$.

Preferences Individuals have preferences over decisions represented by a utility function $v_i(s, \beta_i) : \{0, 1\} \times \{1, \bar{\beta}\} \to \{0, 1, \bar{\beta}\}$. The function v_i returns a value $\beta_i \in \{1, \bar{\beta}\}$ when $s = \tau_i$, zero otherwise, where τ_i describes the voter's preference over policy.

Information Utility functions v and policy preference τ are common knowledge to all voters and candidates. Intensity type β is private information to each voter i. Prior beliefs about β_i are common knowledge, $\Pr(\beta_i = \bar{\beta}) = q$, $\Pr(\beta_i = 1) = 1 - q$ for all i.

Actions In Settings 1 and 2, each voter casts one vote for either Candidate A or Candidate B at an election and each candidate proposes a policy platform $s_p \in \{0, 1\}$, $p \in \{A, B\}$. In Setting 2, each voter selects a magnitude of costly political action $\lambda_i \in \mathbb{R}_+$ of immediate net cost. In Setting 3, each voter sends a message $m_i \in \{1, \bar{\beta}\}$, $m \equiv (m_1, m_2, m_3)$.

Transfer function In Setting 3, in order to provide incentives to make efficient choices, it might be necessary to tax or subsidize one or more of the voters. A transfer function $t : m \to \mathbb{R}^3$ maps the three messages m into a vector of transfers.

Welfare Welfare for voter i, w_i is $v_i(s, \beta_i) - \lambda_i - t_i$. Social welfare, W, is the sum of individual welfare, $\sum_i w_i$.

Mechanism A mechanism is a pair of functions $(f(m), t(m))$, $f : \{1, \bar{\beta}\}^3 \to \{0, 1\}$, $t : \{1, \bar{\beta}\}^3 \to \mathbb{R}^3$ that maps vectors of voter messages into a social decision and vector of transfers.

VCG mechanism A VCG mechanism is a dominant strategy incentive compatible design that returns a social choice that maximizes policy welfare. Each voter's transfer is equal to their social cost such that, if that voter is pivotal in the social choice their transfer is non-positive and if that voter is not pivotal, their transfer is zero.

H.2 Proof to Proposition 4

Proof. **Setting 1** In Setting 1, candidate strategies are independent of voter signals λ and voter strategies σ_V are independent of voter intensities β.

Lemma 1 shows that in equilibrium $s^* = 0$ when $\hat{\beta}_1 + \hat{\beta}_2 \leq \hat{\beta}_3$, else $s^* = 1$. In Setting 1, candidate beliefs about β follow from q. $\beta_1 + \beta_2 \leq \beta_3$ only obtains when $\beta = \{1, 1, \bar{\beta}\}$, which occurs with probability $(1-q)(1-q)(q)$. Thus, with probability $1-(1-q)(1-q)(q)$ the candidates' best response is $s^* = 1$. As $1-(1-q)(1-q)(q)$ is strictly greater than $(1-q)(1-q)(q)$ for all $q \in (0, 1)$, the pure strategy equilibrium in Setting 1 is $s^*_A = s^*_B = 1$.

Social welfare when $s^*_A = s^*_B = 1$ depends upon the distribution of intensities for Voters 1 and 2. With probability q^2, both are high-intensity and get $\bar{\beta}$. With probability $q(1-q) + (1-q)q$ one is high- and the other low-intensity, and with probability $(1-q)^2$ both are low-intensity. This generates social welfare in Setting 1 of

$$\begin{aligned}
W &= q^2(2\bar{\beta}) + 2q(1-q)(\bar{\beta}+1) + 2(1-q)^2(1), \\
&= q^2(2\bar{\beta}) + (2q)(\bar{\beta}+1) - (2q^2)(\bar{\beta}+1) + 2(1-2q+q^2), \\
&= 2q\bar{\beta} - 2q + 2.
\end{aligned} \tag{A12}$$

Setting 2 Setting 2 is the separating equilibrium of Proposition 2. Voters 1 and 2 choose $\lambda_i = \lambda^*$ when $\beta_i = \bar{\beta}$, else $\lambda_i = 0$. Whenever either incurs costly action λ^*, $s^* = 1$. Welfare is

$$\begin{aligned}
w_1 &= (q)v_1(s=1, \beta_1 = \bar{\beta}) + (1-q)v_1(s=1, \beta_1 = 1), \\
&= \underbrace{(q)(\bar{\beta}-\lambda^*)}_{\beta_1=\bar{\beta}} + \underbrace{(1-q)(q)(1)}_{\beta_2=\bar{\beta}} + \underbrace{(1-q)(1-q)(1-q)(1)}_{\beta=\{1,1,1\}} + \underbrace{(1-q)(1-q)(q)(0)}_{\beta=\{1,1,\bar{\beta}\}}, \\
w_2 &= (q)v_2(s=1, \beta_2 = \bar{\beta}) + (1-q)v_2(s=1, \beta_2 = 1). \\
w_1 + w_2 &= 2(q)(\bar{\beta}-\lambda^*) + 2(1-q)(q)(1) + 2(1-q)(1-q)(1-q)(1), \\
&= 2q\bar{\beta} - 2q\lambda^* + 2q - 2q^2 + 2(1 - 2q + q^2 - q + 2q^2 - q^3), \\
&= 2 + 2q\bar{\beta} - 2q\lambda^* - 4q + 4q^2 - 2q^3.
\end{aligned}$$

Voter 3 chooses $\lambda_3 = \lambda^*$ when $\beta_3 = \bar{\beta}$, else $\lambda_3 = 0$. When $\lambda_3 = \lambda^*$, $s^* = 0$ with probability $(1-q)^2$ and $s^* = 1$ with probability $1-(1-q)^2$. Welfare for Voter 3 in Setting 2 is

$$\begin{aligned}
w_3 &= (q)(1-q)^2 v_1(s=0, \beta_3 = \bar{\beta}) + (q)(1-(1-q)^2)v_1(s=1, \beta_3 = \bar{\beta}) + (1-q)(v_1(s=1, \beta_1 = 1), \\
&= (q)(1-q)^2(\bar{\beta}-\lambda^*) + (q)(1-(1-q)^2)(0-\lambda^*), \\
&= q(1-q)^2(\bar{\beta}) - q\lambda^*.
\end{aligned}$$

Social welfare under Setting 2 is

$$\begin{aligned} W &= 2 + 2q\bar{\beta} - 2q\lambda^* - 4q + 4q^2 - 2q^3 + q(1-q)^2(\bar{\beta}) - q\lambda^*, \\ &= 2 + (2q + q(1-q)^2)(\bar{\beta}) - 3q\lambda^* - 4q + 4q^2 - 2q^3. \end{aligned} \tag{A13}$$

Setting 3 Setting 3 implements a VCG mechanism to determine policy s^* and transfers t. The VCG mechanism assigns policy to maximize social welfare,

$$f(m) = \underset{s \in \{0,1\}}{\operatorname{argmax}} \sum_{i=1}^{3} w_i(s), \tag{A14}$$

and assigns transfers to each voter equal to that voter's social cost to the other voters,

$$t_i(m) = \sum_{j \neq i} v_j(m_j, s^*) - \sum_{j \neq i} v_j(m_j, s^*_{-i}), \tag{A15}$$

where s^* is the equilibrium policy returned by f(m) when voter i is part of the electorate and s^*_{-i} is the equilibrium policy were voter i not part of the electorate. (A15) assigns to each voter the lost welfare to other voters had that voter not been part of the electorate, i.e., the social cost of that individual. For a voter who is not pivotal to determining policy, their social cost is zero and they pay no transfers.

The VCG mechanism charges the voter who obtains their preferred policy the transfer cost (A15). Assume if social welfare is equivalent between $s = 0$ and $s = 1$, the mechanism assigns policy with the majority $s^* = 1$ with probability one. This simplifies the analysis but does not change the welfare result.

A VCG mechanism induces a dominant strategy for each player to report a message m_i equal to their private value for policy (Clarke, 1971), in this case intensity β_i. Voters report $m_i = \bar{\beta}$ when high-intensity, and $m_i = 1$ when low-intensity and incur the assigned transfer when policy is set with their preference.

There are eight possible combinations of intensities, $\{1, \bar{\beta}\}^3$, each with a specific probability and a social choice and transfer vector returned by the VCG mechanism. I summarize each of these eight outcomes in Table A1. Columns one, two, and three present the intensities of each voter in each outcome, column four the probability of that outcome, column five the identity of the pivotal voter if there is one, column six the social choice returned by the VCG mechanism, column seven the net utility from policy given the social choice, column eight the transfers paid by the pivotal voter as applicable, and column nine the social welfare of that outcome.

Social welfare in Setting 3 is the average of column nine weighted by the probabilities

Table A1: VCG mechanism outcomes

Voter intensities: Voter 1	Voter 2	Voter 3	Probability	Pivotal voter	$s^* = f(m)$	$\sum_i v_i(s^*)$	$\sum_i t_i(s^*)$	W
1	1	1	(1 – q)(1 – q)(1 – q)		1	1 + 1		2
$\bar{\beta}$	1	1	(q)(1 – q)(1 – q)		1	$\bar{\beta}$ + 1		$\bar{\beta}$ + 1
$\bar{\beta}$	$\bar{\beta}$	1	(q)(q)(1 – q)		1	$\bar{\beta} + \bar{\beta}$		$2\bar{\beta}$
1	$\bar{\beta}$	1	(1 – q)(q)(1 – q)		1	1 + $\bar{\beta}$		$\bar{\beta}$ + 1
$\bar{\beta}$	1	$\bar{\beta}$	(q)(1 – q)(q)	1	1	$\bar{\beta}$ + 1	$\bar{\beta}$ – 1	2
1	$\bar{\beta}$	$\bar{\beta}$	(1 – q)(q)(q)	2	1	1 + $\bar{\beta}$	$\bar{\beta}$ – 1	2
1	1	$\bar{\beta}$	(1 – q)(1 – q)(q)	3	0	$\bar{\beta}$	1 + 1	$\bar{\beta}$ – 2
$\bar{\beta}$	$\bar{\beta}$	$\bar{\beta}$	(q)(q)(q)		1	$\bar{\beta} + \bar{\beta}$		$2\bar{\beta}$

in column four,

$$
\begin{aligned}
W &= 2(1-q)^3 + (\bar{\beta}+1)(q)(1-q)^2 + 2\bar{\beta}(q^2)(1-q) + (\bar{\beta}+1)(q)(1-q)^2 + 2(q^2)(1-q) \\
&\quad + 2(q^2)(1-q) + (\bar{\beta}-2)(q)(1-q)^2 + 2\bar{\beta}(q^3), \\
&= 2(1-q)^3 + (3\bar{\beta})(q)(1-q)^2 + (2\bar{\beta}+4)(q^2)(1-q) + 2\bar{\beta}(q^3), \\
&= (3\bar{\beta}-6)q^3 + (10-4\bar{\beta})q^2 + (3\bar{\beta}-6)q + 2. \qquad \text{(A16)}
\end{aligned}
$$

Welfare comparison The VCG mechanism (Setting 3) produces greater social welfare than no-communication electoral competition (Setting 1) when

$$
\begin{aligned}
(3\bar{\beta}-6)q^3 + (10-4\bar{\beta})q^2 + (3\bar{\beta}-6)q + 2 &> 2q\bar{\beta} - 2q + 2, \\
(3\bar{\beta}-6)q^3 + (10-4\bar{\beta})q^2 + (\bar{\beta}-4)q &> 0, \\
(3\bar{\beta})q^2 - (4\bar{\beta})q + \bar{\beta} &> 6q^2 - 10q + 4, \\
(\bar{\beta})(3q-1)(q-1) &> 2(3q-2)(q-1), \\
(\bar{\beta}/2)(3q-1) &< (3q-2),
\end{aligned}
$$

which never holds because $(3q-1) > (3q-2)$, $q \in (0,1)$, and $(\bar{\beta}/2) > 1$. No communication is always welfare-enhancing over VCG in this setting.

The VCG mechanism (Setting 3) produces greater social welfare than electoral competition with costly political action (Setting 2) when

$$
\begin{aligned}
&(3\bar{\beta} - 6)q^3 + (10 - 4\bar{\beta})q^2 + (3\bar{\beta} - 6)q + 2 \\
&> 2 + (2q + q(1-q)^2)(\bar{\beta}) - 3q\lambda^* - 4q + 4q^2 - 2q^3, \\
&(3\bar{\beta})q^3 - 6q^3 + 10q^2 - (4\bar{\beta})q^2 + (3\bar{\beta})q - 6q \\
&> (2q)(\bar{\beta}) + q(1 - 2q + q^2)(\bar{\beta}) - 3q\lambda^* - 4q + 4q^2 - 2q^3, \\
&2q^2\bar{\beta} - 4q^2 + 6q - 2q\bar{\beta} - 2 > -3\lambda^*, \\
&\bar{\beta}(2q)(q-1) > 2(2q-1)(q-1) - 3\lambda^*, \\
&\bar{\beta}(q) + 3\lambda^*/(2(q-1)) < (2q-1), \\
&(\bar{\beta} - 2)(q) + 3\lambda^*/(2(q-1)) + 1 < 0. \qquad \text{(A17)}
\end{aligned}
$$

Suppose λ^* follows its (intuitive) lower bound from Proposition 2, providing the easiest case for the left hand side to be less than zero. The lower bound of λ^* is $(1-q)^2$ when $q < 1/2$ and $1 - q - (1-q)^2$ otherwise (Proposition 2).

If $q < 1/2$, substituting $(1-q)^2$ for λ^* in (A17), Setting 3 produces greater welfare than Setting 2 when

$$
\begin{aligned}
&(\bar{\beta} - 2)(q) + 3(1-q)^2/(2(q-1)) + 1 < 0, \\
&(\bar{\beta})(q) + q/2 + 1/2 < 0, \\
&(\bar{\beta})(q) < -1/2 - q/2,
\end{aligned}
$$

which can never hold because $\bar{\beta} > 2$ and $q > 0$. Setting 3 is not welfare-enhancing when $q < 1/2$.

If $q \geq 1/2$, substituting $1 - q - (1-q)^2$ for λ^* in (A17), Setting 3 produces greater welfare than Setting 2 when

$$
\begin{aligned}
&(\bar{\beta} - 2)(q) + 3(1 - q - (1-q)^2)/(2(q-1)) + 1 < 0, \\
&(\bar{\beta} - 2)(q) + 3(q)(1-q)/(2(q-1)) + 1 < 0, \\
&(\bar{\beta})(q) < q/2 - 1, \\
&\bar{\beta} < 1/2 - 1/q,
\end{aligned}
$$

which never holds because $1/q < 1$ when $q \in (1/2, 1)$ and $\bar{\beta} > 2$. Setting 3 is not welfare-enhancing when $q \geq 1/2$.

Electoral competition with costly political action (Setting 2) produces greater social welfare than no-communication electoral competition (Setting 1) when

$$
\begin{aligned}
&2 + (2q + q(1-q)^2)(\bar{\beta}) - 3q\lambda^* - 4q + 4q^2 - 2q^3 \\
&> 2q\bar{\beta} - 2q + 2, \\
&(2 + (1-q)^2)(\bar{\beta}) - 3\lambda^* - 2 + 4q - 2q^2 > 2\bar{\beta}, \\
&(1-q)^2(\bar{\beta}) - 2(1 - 2q + q^2) > 3\lambda^*, \\
&\lambda^* < (1-q)^2(\bar{\beta} - 2)/3. \qquad \text{(A18)}
\end{aligned}
$$

Suppose λ^* follows its (intuitive) lower bound from Proposition 2, providing the easiest case for the left hand side to be less than the right. The lower bound of λ^* is $(1 - q)^2$ when $q < 1/2$ and $1 - q - (1 - q)^2$ otherwise (Proposition 2).

If $q < 1/2$, substituting $(1 - q)^2$ for λ^* in (A18), Setting 2 produces greater welfare than Setting 1 when

$$\begin{aligned}&(1 - q)^2 < (1 - q)^2(\bar{\beta} - 2)/3,\\ &1 < (\bar{\beta} - 2)/3,\end{aligned}$$

which holds if and only if $\bar{\beta} > 5$.

If $q \geq 1/2$, substituting $1 - q - (1 - q)^2$ for λ^* in (A18), Setting 2 produces greater welfare than Setting 1 when

$$\begin{aligned}&1 - q - (1 - q)^2 < (1 - q)^2(\bar{\beta} - 2)/3,\\ &q - q^2 < (1 - 2q + q^2)(\bar{\beta} - 2)/3,\\ &3q - 3q^2 < (1 - 2q + q^2)(\bar{\beta}) - 2 + 4q - 2q^2,\\ &2 - q - q^2 < (1 - 2q + q^2)(\bar{\beta}),\\ &(2 + q)/(1 - q) < (\bar{\beta}),\end{aligned}$$

which holds if and only if $\bar{\beta} > 5$ and requires increasing $\bar{\beta}$ with increasing q.

Setting 2 produces greater welfare than Setting 1 at all q when $\bar{\beta} > 5$. Both electoral competition settings (1 and 2) produce greater welfare than VCG (Setting 3) at all q. $\square$

I Proposition A1: Even split on policy

To show that an equilibrium of costly political action does not depend on strategic considerations surrounding a policy minority, I present a version of the model with an even split in the electorate. Consider, contrary to the assumptions on $\tau \equiv (\tau_1, \tau_2, \tau_3)$ in the main model, a split in electorate support for s = 0 and s = 1 by setting β_2 to zero, common knowledge. The electorate is divided into one third preferring s = 1, one third voting solely on its election shock ($\beta_2 = 0$), and one third preferring s = 0. Intensities β_1 and β_3 remain private knowledge up to q.

Proposition A1 (Choice of costly action with a split electorate)**.** *With an even split in the electorate between those on two sides of a policy, a separating equilibrium exists where high-intensity voters ($\beta_i = \bar{\beta}$) choose political action $\lambda_i = \lambda^* > 0$ and low-intensity voters ($\beta_i =$ 1) choose $\lambda_i = 0$, revealing that voter(s) with $\lambda_i = \lambda^*$ are high-intensity. The two candidates converge to offer the policy preferred by high-intensity voter(s) with political action λ^*.*

Proof. Begin by specifying expected votes for each candidate. Let the candidates' estimates of the intensity of Voters 1 and 3 after observing λ_1 and λ_3 be $\hat{\beta}_1$ and $\hat{\beta}_3$. Given the probability that each voter prefers Candidate A over B and the uniform distribution on δ, expected vote count for Candidate A is

$$\begin{aligned} V^A &= \frac{\hat{\beta}_1(s_A - s_B) - c}{d - c} - \frac{c}{d - c} + \frac{\hat{\beta}_3(s_B - s_A) - c}{d - c} \\ &= (\hat{\beta}_1(s_A - s_B) + \hat{\beta}_3(s_B - s_A) - 3c)/(d - c) \end{aligned}$$

with $V^B = 3 - V^A$.

Candidate A's best response to $s_B = 0$ is $s_A = 0$ when

$$\begin{aligned} V^A(0|s_B = 0) &\geq V^A(1|s_B = 0), \\ -3c/(d - c) &\geq (\hat{\beta}_1 - \hat{\beta}_3)/(d - c) - 3c/(d - c) \Rightarrow \hat{\beta}_3 \geq \hat{\beta}_1. \end{aligned}$$

Candidate A's best response to $s_B = 1$ is $s_A = 0$ when

$$\begin{aligned} V^A(0|s_B = 1) &\geq V^A(1|s_B = 1), \\ (-\hat{\beta}_1 + \hat{\beta}_3)/(d - c) - 3c/(d - c) &\geq -3c/(d - c) \Rightarrow \hat{\beta}_3 \geq \hat{\beta}_1. \end{aligned}$$

Likewise, Candidate B's best response to $s_A = 0$ is $s_B = 0$ when

$$\begin{aligned} V^B(0|s_A = 0) &\geq V^B(1|s_A = 0), \\ 3 + 3c/(d - c) &\geq 3 - (-\hat{\beta}_1 + \hat{\beta}_3)/(d - c) + 3c/(d - c), \\ 0 &\geq (\hat{\beta}_1 - \hat{\beta}_3)/(d - c) \Rightarrow \hat{\beta}_3 \geq \hat{\beta}_1. \end{aligned}$$

Candidate B's best response to $s_A = 1$ is $s_B = 0$ when

$$\begin{aligned} V^B(0|s_A = 1) &\geq V^B(1|s_A = 1), \\ 3 - (\hat{\beta}_1 - \hat{\beta}_3)/(d - c) + 3c/(d - c) &\geq 3 + 3c/(d - c), \\ (-\hat{\beta}_1 + \hat{\beta}_3)/(d - c) &\geq 0 \Rightarrow \hat{\beta}_3 \geq \hat{\beta}_1. \end{aligned}$$

Both candidates propose s = 0 if and only if $\hat{\beta}_3 \geq \hat{\beta}_1$ else s = 1.

Suppose a separating equilibrium exists where when $\beta_i = \bar{\beta}$, $\lambda_i = \lambda^* > 0$, and when $\beta_i = 1$, $\lambda_i = 0$. Then, it must also be that

$$\begin{aligned} V^A(s_A = \tau_i|\lambda_i = \lambda^*) &\geq V^A(s_A = 1 - \tau_i|\lambda_i = \lambda^*), \text{ and} \\ V^B(s_B = \tau_i|\lambda_i = \lambda^*) &\geq V^B(s_B = 1 - \tau_i|\lambda_i = \lambda^*), \text{ and} \\ U^i(\lambda^*|\beta_i = \bar{\beta}) &\geq U^i(0|\beta_i = \bar{\beta}), \text{ and} \\ U^i(0|\beta_i = 1) &\geq U^i(\lambda^*|\beta_i = 1). \end{aligned}$$

The first two inequalities hold. When one voter is revealed high-intensity through choice of political action, the candidates' best responses are to set policy at that voter's preference. When both voters reveal high- or low-intensity through choice of action, the candidates are indifferent over choice of policy.

Continuing with the third and fourth inequalities for Voter i, consider Voter 3 with $\tau_3 = 0$ but with symmetry to Voter 1. Voter 3 expects Voter 1 in the separating equilibrium to be playing $\lambda_1 = \lambda^*$ with probability q and $\lambda_1 = 0$ with probability 1 – q. Voter 3's expected benefit from $\lambda_3 = \lambda^*$ and $\lambda_1 = 0$ is

$$\begin{aligned} U^3(\lambda^*|\beta_3) &= qU^3(\lambda^*|\hat{\beta}_1 = \bar{\beta}, \beta_3) + (1 - q)U^3(\lambda^*|\hat{\beta}_1 = 1, \beta_3), \\ U^3(0|\beta_3) &= qU^3(0|\hat{\beta}_1 = \bar{\beta}, \beta_3) + (1 - q)U^3(0|\hat{\beta}_1 = 1, \beta_3). \end{aligned}$$

Assume that the candidates randomize s when they are indifferent, i.e. when $\hat{\beta}_1 = \hat{\beta}_3$. Then, given the candidate best response functions to λ_1 and λ_3,

$$\begin{aligned} U^3(\lambda^*|\hat{\beta}_1 = \bar{\beta}, \beta_3 = \bar{\beta}) &= \bar{\beta}/2 - \lambda^*, \\ U^3(\lambda^*|\hat{\beta}_1 = 1, \beta_3 = \bar{\beta}) &= \bar{\beta} - \lambda^*, \\ U^3(0|\hat{\beta}_1 = \bar{\beta}, \beta_3 = \bar{\beta}) &= 0, \\ U^3(0|\hat{\beta}_1 = 1, \beta_3 = \bar{\beta}) &= \bar{\beta}/2, \\ \\ U^3(\lambda^*|\hat{\beta}_1 = \bar{\beta}, \beta_3 = 1) &= 1/2 - \lambda^*, \\ U^3(\lambda^*|\hat{\beta}_1 = 1, \beta_3 = 1) &= 1 - \lambda^*, \\ U^3(0|\hat{\beta}_1 = \bar{\beta}, \beta_3 = 1) &= 0, \\ U^3(0|\hat{\beta}_1 = 1, \beta_3 = 1) &= 1/2. \end{aligned}$$

The first inequality $U^3(\lambda^*|\beta_3 = \bar{\beta}) \geq U^3(0|\beta_3 = \bar{\beta})$ holds when

$$
\begin{aligned}
&q[U^3(\lambda^*|\hat{\beta}_1 = \bar{\beta}, \beta_3 = \bar{\beta})] + [1 - q][U^3(\lambda^*|\hat{\beta}_1 = 1, \beta_3 = \bar{\beta})] \geq \\
&q[U^3(0|\hat{\beta}_1 = \bar{\beta}, \beta_3 = \bar{\beta})] + [1 - q][U^3(0|\hat{\beta}_1 = 1, \beta_3 = \bar{\beta})], \\
&q[\bar{\beta}/2 - \lambda^*] + [1 - q][\bar{\beta} - \lambda^*] \geq q[0] + [1 - q][\bar{\beta}/2], \\
&q\bar{\beta}/2 + \bar{\beta} - \lambda^* - q\bar{\beta} \geq \bar{\beta}/2 - q\bar{\beta}/2, \\
&\bar{\beta} - \bar{\beta}/2 \geq \lambda^*, \\
&\bar{\beta}/2 \geq \lambda^*.
\end{aligned}
$$

The second inequality $U^3(0|\beta_3 = 1) \geq U^3(\lambda^*|\beta_3 = 1)$ holds when

$$
\begin{aligned}
&q[U^3(0|\hat{\beta}_1 = \bar{\beta}, \beta_3 = 1)] + [1 - q][U^3(0|\hat{\beta}_1 = 1, \beta_3 = 1)] \geq \\
&q[U^3(\lambda^*|\hat{\beta}_1 = \bar{\beta}, \beta_3 = 1)] + [1 - q][U^3(\lambda^*|\hat{\beta}_1 = 1, \beta_3 = 1)], \\
&q[0] + [1 - q][1/2] \geq q[1/2 - \lambda^*] + [1 - q][1 - \lambda^*], \\
&1/2 \geq 1 - \lambda^*, \\
&\lambda^* \geq 1/2.
\end{aligned}
$$

Therefore, a separating equilibrium exists when $1/2 \leq \lambda^* \leq \bar{\beta}/2$. □

With equal shares of preference-0 and preference-1 voters, the two candidates converge to offer the policy they expect to yield the high-intensity voter(s). When they believe Voter 1 and Voter 3 have the same intensity for policy, high or low, either policy platform pair $s_A = s_B = 0$ or $s_A = s_B = 1$ can be supported in equilibrium. However, when they believe Voter 1 (Voter 3) is the only high-intensity voter, they propose policy $s = 1$ ($s = 0$). This provides an incentive for voters to communicate to candidates that they are high-intensity types and to choose costly political action in equilibrium.