# On-line Appendix <br> A Disconnect in Representation? Comparison of Trends in Congressional and Public Polarization Journal of Politics, 2015 

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## A Number of questions and responses per year

In Table A1, we present the number of items and number of non-missing responses used in our joint scaling for each year and by the partisan identification of the respondent. For example, in 1956 we scaled 10 items and observed 8,317 non-missing responses to those 10 items among Democratic identifiers (including leaners), 6,187 responses from Republican identifiers (including leaners), and 2,097 among independents.

Table A1: Number of questions and responses per year

| Year | Number questions | Democrat | Republican | Independent |
| :---: | :---: | :---: | :---: | :---: |
| 1956 | 10 | 8,317 | 6,187 | 2,097 |
| 1958 | 5 | 3,679 | 2,235 | 746 |
| 1960 | 7 | 3,987 | 2,655 | 862 |
| 1962 | 3 | 1,805 | 1,168 | 405 |
| 1964 | 13 | 10,706 | 5,338 | 1,668 |
| 1966 | 5 | 2,930 | 1,703 | 711 |
| 1968 | 11 | 7,695 | 4,588 | 1,626 |
| 1970 | 9 | 5,490 | 3,159 | 1,330 |
| 1972 | 16 | 16,402 | 11,066 | 4,554 |
| 1974 | 6 | 4,072 | 2,656 | 1,352 |
| 1976 | 15 | 13,476 | 9,299 | 4,156 |
| 1978 | 10 | 9,900 | 5,828 | 3,001 |
| 1980 | 12 | 7,708 | 5,256 | 2,051 |
| 1982 | 6 | 3,947 | 2,420 | 943 |
| 1984 | 22 | 17,793 | 15,422 | 4,742 |
| 1986 | 22 | 19,364 | 13,895 | 5,337 |
| 1988 | 28 | 23,453 | 20,792 | 5,771 |
| 1990 | 27 | 19,978 | 14,457 | 4,570 |
| 1992 | 40 | 43,673 | 33,338 | 11,183 |
| 1994 | 25 | 19,787 | 17,989 | 4,501 |
| 1996 | 32 | 26,445 | 19,558 | 4,560 |
| 1998 | 14 | 8,737 | 6,283 | 2,019 |
| 2000 | 35 | 24,932 | 19,410 | 6,411 |
| 2002 | 12 | 6,562 | 6,497 | 1,293 |
| 2004 | 30 | 15,706 | 13,377 | 3,588 |
| 2008 | 31 | 34,910 | 17,024 | 7,488 |
| 2012 | 28 | 31,403 | 14,419 | 5,736 |
| Total | 67 | 392,857 | 276,019 | 92,701 |

Note: Cell entries for columns three, four, and five are the number of non-missing choices observed from respondents of that partisan identification in that survey year.

## B Ten item analysis, 1984-2008

In this section, we estimate ideology from a set of 10 policy questions that were asked consistently across ten surveys in the 1984 to 2008 biennial time series of the American National Election Studies (ANES). Although we believe that the most informative use of our data is to include all of the information we have, this exercise allows us to confirm that using different numbers or types of items in different years does not lead to divergent results. Past work on polarization typically uses consistent sets of items over time at the expense of richer and more encompassing data.

The data set used here consists of ten policy questions asked in each of ten different releases of the Study, 1984 to 2008 1 In Figure 2 in the main body, we present the marginals over time for each of these ten questions. We estimate ideology for this more limited data set using the same procedure we use for the full data set described in the main body.

In Figure A1, we plot the mean posterior standard deviation with a $95 \%$ credible interval for each year. As noted in the body of the paper, the scale of ideology is arbitrary, so the standard deviation of all ideal points across all years is set to 1 , and the standard deviation in each individual year is identified relative to the grand mean. If variance in preferences were increasing from 1984 to 2008 , we should see standard deviations below 1 in the early years, and standard deviations above 1 later in the series.

Figure A1 confirms the results we obtained using the full data set. The estimated standard deviation changes little from 1984 to 2008. One way to analyze the data is to look at just the endpoints. In 1984, the estimated standard deviation is almost exactly equal to the grand mean. In 2008, the standard deviation is above the grand mean by about $1 \%$. The difference is not statistically significant, with large overlap in the two credible intervals. Overall, all but two years cannot be differentiated from the grand mean. Substantively, there is no estimated divergence from the grand mean as great as $5 \%$.

As in the full-data analysis, the estimated fluctuations do not appear to follow a clear over-time trend. Both the year of lowest standard deviation and the year of highest standard deviation occur in the middle of this time period. 1992 is the low point, with about $4 \%$ less variation, and 1996 is the high point, with about $2.5 \%$ more variation. 1996 is also the only year in which we can reject the hypothesis that the standard deviation is less than 1.

[^1]Figure A1: Dispersion of the estimated distributions over time from the 10-item analysis


Note: Each point is the standard deviation of the estimated ideal points in that year from the constant 10-item dataset.

## C Histograms of point estimates and posteriors of public ideology

In Figure A2, we present densities of the ideal point estimates for each year using an estimator we believe does not accurately measure polarization. In each year, we calculate the posterior median ideal point for each respondent in that year's survey, and then construct a histogram to summarize the distribution of these ideal points. These graphs demonstrate a clear pattern. In the early years of the ANES, estimated ideal points are clustered around zero. In later years, estimates spread out substantially and obtain a roughly normal distribution. However, the reason for the apparent increase in spread is not necessarily increasing polarization. Point estimates cluster around 0 in 1958 because little is known about the political positions of those respondents. We only have five questions related to ideology from that survey. Our prior has a mean of zero, and while the questions asked in the early years do discriminate somewhat between different ideal points, the likelihood is not strong enough to yield ideal points distant from 0 . If a voter tells us that the government should ensure fair jobs and housing for blacks, we learn, albeit with some uncertainty, that the voter is not a staunch conservative in 1958. But we learn little about exactly how liberal that person is.

To see why Figure A2 is more about the number of items and their informativeness in each year than about polarization, consider Figure A3. To draw conclusions about the distribution of preferences, we must account for uncertainty in our estimates of those preferences. One simple way to evaluate our posterior beliefs about the entire distribution of ideal points is to create a density of densities. That is, in Figure A3 we plot histograms all of the samples from the posterior distribution of all of the ideal points. These histograms reflect not only the posterior medians, as are plotted in Figure A2, but also the posterior uncertainty.

The conclusion from Figure A3 is different from Figure A2. Here we find that the estimated distributions look quite similar over time, especially for the early years in the time series in the first row. One might be tempted to think that these posteriors are reflections of the normally distributed prior. We know this is not the case by looking back at Figure A2. If our data contained little information about the ideal points, then all of the point estimates would tend to be clustered towards 0. What occurs in Figure A3, however, is that our estimates of the ideal points show a relatively normal distribution of individuals even in 2012. In 1956, our estimates also look normal, but this is more reflective of our lack of certainty about individual ideal points than certainty that a declining portion of ideal points are at the extremes.

The distinction between these two estimators is highlighted with Figure 3 in the main body.
Note: The height of the bars in these histograms represent the proportion of respondents whose median posterior ideal point falls within that bin; $x$ - and y-axes are scaled to the same range so that heights may be compared across figures, with higher bars in the middle meaning less spread. Changes in the distributions of point estimates reflect two major factors. First, they may reflect different underlying distributions. Secondly, they may reflect different uncertainties over individual positions. Changes in this graph are almost entirely driven by the latter factor.
Figure A3: Estimated densities of policy views, 1956-2012

Note: The height of the bars in these histograms represent the proportion of all ideal points sampled from the posterior distribution
 higher bars in the middle meaning less spread. The distributions in these figures reflect posterior beliefs about the entire distribution of individual preferences in each year, including uncertainty due to variation in the number and discrimination of items.

## D Power analysis

We test the statistical power of our approach to estimating polarization using 10 items over time in the ANES using a monte carlo simulation. We simulated 100 data sets of 4,000 respondents and their responses to the 10 policy items we use in Section Babove. To simulate the $4,000,000$ responses, we first sampled ideal points for the 4,000 respondents in each simulation. We then used the posterior point estimates of the item parameters from our model for the 10 items along with the multinomial IRT model to generate item responses. That is, to generate the data, the ideal points for the respondents vary across simulations but the item parameters stay constant across simulations. We then ran our model on each of the 100 data sets of observed item responses to recover 100 estimates of item parameters and ideal points.

To test the ability of our model to identify polarization, we drew ideal points for the 4,000 respondents from two different normal distributions. In each simulation, the ideal points for 2,000 respondents were drawn from a standard normal distribution with mean 0 and standard deviation 1. The ideal points for the other 2,000 respondents were drawn from a normal distribution with mean 0 and standard deviation 1.1. Thus the latter 2,000 respondents were more polarized (on average) because the standard deviation of their ideal points was 10 percent larger. We chose the sample size 2,000 because the median number of respondents to our set of 1984-2008 ANES studies is 2,010 . These simulations test to see if we can detect an increase in polarization (as defined by standard deviation) of 10 percent between just two election studies. We believe this to be somewhat conservative, because our actual data span many election studies, and so we get to compare more than just two years. Note also that the simulation incorporates the sampling error present in each of the real ANES surveys: although the ideal points are sampled from two distributions with different standard deviations, it is unlikely that in any simulation the 4,000 sampled ideal points are exactly 10 percent different in their spread.

We estimated the model on each of the 100 simulations (which took a few weeks to run) and in each simulation tested the hypothesis that the ratio of the standard deviations of the second 2,000 ideal points to the standard deviation of the first 2,000 ideal points was greater than 1. In 90 of 100 samples we accept the hypothesis that the ratio is greater than 1 at $p<.05$, and in 92 at $p<.1$. Even with this conservative test, comparing only two years worth of data with an increase of spread of only 10 percent, we detect the effect in 9 out of 10 simulations. This suggests our estimator has enough statistical power to avoid Type II errors at standard levels of significance.

Interestingly, we do find a systematic underestimate of the ratio: the median across the one hundred simulations of the posterior mean of the ratio is 1.05 , while the true value is 1.1 . This suggests one drawback to using only 10 items is that we are not as able to spread individuals out as might be desirable. The analysis in the main body of the paper with many more items and years should have greater statistical power to detect polarization.

## E Standard deviations of ideal points for college educated and those who have tried to influence others

Our goal is to examine polarization of the public at large. However, a portion of the existing work on polarization places special importance on activists and other politically involved citizens (e.g., Abramowitz, 2010). Unfortunately, the American National Election studies do not have large enough sample sizes to study the most active citizens with any degree of precision. However, as a starting place for future inquiry, Figure A4 shows the standard deviation of ideal points for citizens who hold a college degree or higher level of education, and citizens who say they tried to influence the votes of others during the most recent election, respectively. These correspond to ANES cumulative file variable numbers VCF0110 and VCF0717. 17\% of respondents for the entire period held a college degree or higher, and $26 \%$ reported trying to influence the votes of others. We caution that this is a substantial reduction in overall sample size that varies by year.

Figure A4 does not show a clear over time pattern. The average standard deviation is higher in the latter half of the data than for the first half, perhaps suggesting more polarization than for the population as a whole. However, we note that this is a compositional effect. The proportion of the population that is college educated, in particular, grows substantially over this period. Given the lack of polarization in the overall population, any increase in polarization among the college educated is offset by a decrease in polarization in the rest of the population. Frame (b) of Figure A4 presents suggestive evidence that the more activist segment of the population has increasingly disperse policy views over this time period. The amount of dispersion is about 15 percent, with the standard deviation in more recent years around 1.15 compared to the overall average standard deviation of 1.0.

Figure A4: Standard deviation of ideal points for highly educated and interested citizens, 19562012


Note: Each line represents the 95\% posterior credible interval for the standard deviation of voter policy positions in each year.

## F Presentation of figures from alternative statistical specifications

In this section, we present results from three alternative statistical specifications that address three potential concerns with our main analysis. We present results from an ordered logit specification in Section F.1, an analysis of social items only in Section F.2, and an analysis where the meaning of policy issues is allowed to change over time in Section F.3. We present four frames in each of Figures A5 to A11. These frames reproduce the graphic from the main body in frame (a), and produce the exact same graphic based on the alternative specification in frames (b), (c), and (d).

## F. 1 Replication of full analysis with ordered logit

As we discuss in the main body, one concern with our model is that the multinomial specification has less statistical power than a model that assumes order to the responses to these survey questions, which have a natural order. In this section, we replicate the analysis from the main body using an ordered logit specification for all items in our set of policy questions. We use MCMCpack's (Martin, Quinn, and Park, 2011) MCMCordfactanal function to run the model. Due to memory limitations with the MCMCpack implementation, we were only able to run the model on a $40 \%$ random sample of the full data. We run a 200 iteration burn in and take 2,000 posterior samples, thinning by 2. Frame (b) in Figures A5 to A11 presents the results. To our reading, no conclusions would be changed if we make the stronger assumption for an ordered logit specification relative to our multinomial specification.

## F. 2 Replication of analysis for social issues only

A second concern with our model is that economic issues may be dominating emerging polarization on social issues, which may be an important part of polarization in the public (i.e., the culture war). In this section, we estimate the same statistical model as in the main body using only social policy issue items. Due to the lack of social issue questions from earlier periods, our analysis begins in 1980 for this estimation. From the cumulative ANES file, we include items on school prayer (VCF9043 and VCF9051), rights of the accused (VCF0832), the Equal Rights Amendment (VCF0833), women's equal role (VCF0834), abortion (VCF0837 and VCF0838), gay discrimination (VCF0876a), gays in the military (VCF0877a), and gay adoption of children (VCF0878). We run a 200 iteration burn in and take 2,000 posterior samples, thinning by 2. Frame (c) in Figures A5 to A11 presents the results. We do not find large increases in polarization for social items only. There is some evidence of party sorting (frame [c] of Figure A8) on social issues as with the larger dimension of ideology we estimate in the main body.

## F. 3 Replication of full analysis allowing issue meaning to change over time

In this section, we present results from the same statistical model as in the main body of the paper, but with a recoding of data such that the meaning of policy questions is allowed to change over time. Our main estimation assumes that policy questions asked in the 1950s have the same average relationship to ideology as they do in the 2010s. While this is an important component in allowing the distribution of ideology to be compared over time, it is a potentially strong assumption. In order to relax the assumption while maintaining the comparability of the distribution over time, we allow policy questions to have different meanings over time, limiting the "run" of any policy question to 10 years or less. Specifically, we allow the item parameters for each question to take on different values if the item is observed in the data set more than 10 years apart. We do this by creating "new" items for each question that spans more than 10 years. Substantively, this means
that while previously the same item would have the same relationship to the latent scale of ideology in 1956 as in 2012 (if asked in both years), in this estimation no item is allowed to have the same meaning for longer than 10 years. This allows issue meaning to evolve and change over time.

For example, imagine the ANES asked respondents the same question about the appropriate size of government in 1952, 1954, 1962, 1964, 1980 and 2012. We wrote an algorithm that would split this question into four different questions with respect to how it enters our IRT model. One item would have the same item parameters for the 10 years that include the 1952, 1954, and 1962 versions of the question, and 1964, 1980, and 2012 would each have their own item parameters. From the estimator's perspective, there is no information that ties these items together (other than the information that ties all other items together in the correlation in respondents' responses). A more sophisticated model might do such a thing, e.g. allowing item parameters to change over time as a random walk, but we implemented this simple and stark algorithm for simplicity and transparency.

The R code for our algorithm which we applied to each item in turn is

```
unGlue <- function(x,max.distance) \{
    \# Unglue a sequence of numbers into multiple sequences of distance
    \# less than max.distance.
    \# Arguments.
    \# \(x\) - a sequence of numbers.
    \# max.distance - maximum distance within any of the new sequences.
    \# Returns \(x\) named for which item sequence each element resides.
    breaks <- min(x)-1
    \(x<-\operatorname{sort}(x)\) \# sort
    \(\max . x<-\max (x, n a . r m=T)\)
    breaks <- c(breaks, x[1] + max.distance) \# first break
    \# Maximum value of \(x\) within this break.
    max.in.break \(<-\max (x[x<=\) breaks[length(breaks)]])
    while (max.in.break < max.x) \{
        \# Calculate and append next break.
        next.break \(<-\min (x[x>b r e a k s[l e n g t h(b r e a k s)]])+m a x . d i s t a n c e\)
        breaks <- c (breaks, next.break)
        \# Recalculate max.in.break.
        \# While loop breaks when max.in.break == max.x.
        max.in.break \(<-\max (x[x<=\) breaks[length(breaks)]])
    \}
    names (x) <- cut (x,breaks=breaks,
                                    labels=sprintf("Item \%s",
                                    \(\operatorname{seq}(1\), length (breaks) -1\())\) )
    return (x)
\}
```

We applied the algorithm to each issue question we used in the model presented in the main body. We present a new item map for this modified set of items in Figure A12. In comparison to Figure 1 in the main body, Figure A12 highlights that the duration of each item is limited; dots representing year of coverage do not cover great distances across the x -axis for each row.

In practice, this means that no one item glues the ideological scale together over too long a time period, and preferences over the same policy question are allowed to mean different things over time.

We present new results from the version of our estimation allowing issue meaning to vary over time in Frame (d) in Figures A5 to A11. We run a 200 iteration burn in and take 2,000 posterior samples, thinning by 2 . Comfortingly, relaxing the assumption of long time periods of constant meaning for the set of items that are in the data for longer than a ten year period does not alter our conclusions. Results allowing issues to mean different things over time are consistent and similar to the results we present in the main body.

## References

Abramowitz, A.I. 2010. The Disappearing Center: Engaged Citizens, Polarization, and American Democracy. Yale University Press.

Martin, Andrew D., Kevin M. Quinn, and Jong Hee Park. 2011. "MCMCpack: Markov Chain Monte Carlo in R." Journal of Statistical Software 42(9): 22.

Figure A5: Variance of public ideology measured two different ways


Note: Each frame compares two estimators for the variance of public ideology. The gray squares present the standard deviation of the posterior median ideal point of each respondent, measured in each year of the ANES. The black circles present the median standard deviation, with black line a 95 percent credible interval, of ideal points in each year summarized by considering all posterior iterations. The former measure can lead to inaccurate inference if the number or discrimination of items across years varies, which would not be accounted for by point summaries such as the mean or median. The latter measure accounts for uncertainty in individual estimates. The two measures lead to different conclusions about polarization in the public, with the former suggesting an increase, and the latter suggesting no trend or a slight decrease. Dashed and dotted lines are OLS fits to the two series.

Figure A6: Standard deviation of ideal points, 1956-2012


Note: Each line represents the 95\% posterior credible interval for the standard deviation of voter policy positions in each year.

Figure A7: Replacement by more divergent ideal points, 1956-2012


Note: Each point plots the proportion of respondents or in that year who reside outside the interior 95 percent of the distribution of respondent ideal points from the previous year. With no change in spread, this point on average should take on the value of 0.05, noted by the dashed line. Lines represents the $95 \%$ posterior credible interval.

Figure A8: Party medians, 1956-2012


Note: Each line represents the $95 \%$ posterior credible interval for the median of the ideal points for each group. The squares are Republican medians, the triangles are Independent medians, and the circles are Democratic medians.

Figure A9: Party standard deviations, 1956-2012


Note: Each line represents the 95\% posterior credible interval for the standard deviation of the ideal points for each group. The squares are Republican standard deviations, the triangles are Independent standard deviations, and the circles are Democratic standard deviations.

Figure A10: Party overlap, 1956-2012


Note: Each line represents the $95 \%$ posterior credible interval and posterior median for the proportion of the ideal points in each group that lie in the Democratic-Republican overlap interval. The interval is defined by the space to the right of the 95th percentile of the distribution of Democrats and to the left of the 5th percentile of the distribution of Republicans. The red points are Republican proportions, the black points are the Independent proportions, and the blue points are Democratic proportions. The maximum value this statistic can achieve for partisans is $95 \%$.

Figure A11: Between-party variance, 1956-2012


Note: Each point represents the estimated proportion of variance attributable to between-party variance in each year. Lines are 95\% credible intervals.
Figure A12: Set of questions used for data analysis with issue meaning allowed to change over time



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[^1]:    ${ }^{1}$ There was no study in midterm year 2006, and the 1998 and 2002 studies did not ask all ten of these questions.

